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A Computational Approach to Dynamic Bipedal Walking

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Abstract

The main contribution of this work is a general method for stabilization of periodic orbits for hybrid systems with impact effects. Our primary motivation is controller synthesis for walking robots, but the method can be also applied to problems such as flight control or automotive control. Limit cycles of hybrid systems are characterized by the fact that they span different dynamic regimes. For smooth systems, dynamics of the system along the limit cycle can be decomposed into the transverse and tangential components. We demonstrate that this decomposition can be adapted to hybrid systems. Furthermore, we show that when the transverse dynamics is linearized and discretized, the resulting robust control synthesis problem can be cast as a semidefinite program and thus efficiently solved. We demonstrate our results through the simulation on a simple planar biped robot.

Keywords: hybrid systems, limit cycles, bipedal robots, impact, linear matrix inequalities

1 Introduction

Walking has been extensively studied in the robotics literature. Topics of interest include design, modeling, dynamic analysis, and gait synthesis for walking robots and we refer the reader to monographs [1], [2] and [3] for an overview and further references. The focus of this work is on the design of controllers for dynamically stable bipedal locomotion. The existing methods can be roughly divided into two categories. Those in the first category (e.g. [4, 5, 6]) are based on trajectory tracking. A trajectory (time parameterized curve) is chosen along the orbit and then tracked using any of the available control methods. The drawback of this approach is that it is too rigid since we are typically not interested in tracking the phase (position along the periodic orbit). The methods in the other category use Poincaré sections to compute controller parameters [2, 7, 8, 9]. These methods are limited by the complexity of the computation of the Poincaré section and the choice of the controller structure.

One inherent property of walking robots are switches in the dynamics and impacts. They can be therefore naturally modeled as hybrid systems [10]. The gait of the robot can be represented as a finite automaton, with each state of the automaton corresponding to a different set of dynamic equations. Due to switches in the dynamics, methods designed for smooth systems (e.g. for trajectory tracking) have to be appropriately modified for walking robots. The above references provide some approaches to resolving this problem.

In this paper we propose a novel approach to stabilization of periodic orbits for hybrid systems with impact effects, and thus to design of controllers for walking robots. Periodic orbits of hybrid systems are characterized by the fact that they span different dynamics regimes. As a result, system dynamics along a periodic orbit is not continuous.

In the smooth case, the system dynamics along the periodic orbit can be partitioned into transversal and tangential components and the stabilization of the periodic orbit reduces to the stabilization of the zero solution of the transverse dynamics [11]. The transverse dynamics is in general a periodic time-varying nonlinear system. However, [11] provides the conditions on the linearized version of the transverse dynamics that guarantee the stability of the periodic orbit. The important feature of this approach is that the transversal dynamics can be expressed as a function of the tangential coordinate so the tracking of the phase along the limit cycle can be decoupled from the stabilization of the transversal dynamics. The approach is therefore fundamentally different from the methods that rely on trajectory tracking.

In the case of hybrid systems, the dynamics can still be partitioned into the tangential and transversal components. The linearization along the limit cycle then results in a periodic piecewise linear time-varying system. However, upon discretization, the resulting periodic linear discrete time-varying system is amenable to the recently developed methods for design of robust controllers [12]. The controllers can be efficiently computed by solving a system of linear matrix inequalities. The resulting LMI controller is vastly superior to traditional gain-scheduled designs [13] since the approach directly guarantees the stability of the complete system, no additional assumptions are needed on the system dynamics. An important feature of our approach is that except for choosing the periodic orbit, all the steps can be completely automated. Furthermore, the approach is completely general and can be applied to stabilization of underactuated robots with impact effects in full 3D space (even though we use the example of a fully actuated planar robot in this paper).

Finally, while the method is designed for stabilization of

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a single periodic orbit, adaptability of the robot can be achieved by defining a series of gait primitives. The corresponding stabilizing controllers can be computed off-line and employed as necessary if the robot needs to change its trajectory. See for example [14] for a similar approach in avionics.

The paper is organized as follows. We first review the canonical decomposition of the dynamics along the limit cycle into the tangential and transversal components for smooth autonomous systems. Then we extend this concept to hybrid control systems with impact effects. Subsequently we discuss the discretization of transversal dynamics. Section 3 contains the main results: based on the resulting discretized transversal dynamics, a system of linear matrix inequalities is formulated, from which robust piecewise state feedback controller is obtained. We also address issues about nominal input computation and tangential dynamics control. In section 5 we discuss some implementation aspects. We conclude the paper with an example of a planar bipedal robot and show that the controller designed through our approach stabilizes the robot to the periodic orbit.

2 Decomposition of the dynamics along a periodic orbit

Consider a smooth autonomous dynamical system $\dot{x} = f(x)$ that has a periodic orbit $\eta \subset R^n$ with period T. Let $y(\theta), \theta \in [0,T]$ be a parameterization of η:

$$\frac{dy(\theta)}{d\theta} = f(y(\theta)), \qquad y(0) \in \eta$$

In other words, $y(\theta)$ is a trajectory of the system for an initial condition on η.

The following decomposition of dynamics is motivated by the fact that we are typically only interested in driving the system toward the periodic orbit (so called orbital stability) rather than stabilization of the system toward a particular trajectory on the periodic orbit. In other words, the distance from the periodic orbit should be asymptotically stabilized to 0, but the motion along the orbit can be arbitrary (or is controlled independently). The following briefly summarizes the results in [11].

For x in a neighborhood of η we define the **tangential coordinate**:

$$\theta = \psi_1(x) = \arg\min_{\theta \in [0,T]} \|y(\theta) - x\| \tag{1}$$

Since the periodic orbit is a curve in n-dimensional space, there exist $n-1$ functions ψ_i ($i = 2, \ldots, n$) that are independent and identically vanish on η. We define:

$$\rho_{i-1} = \psi_i, \quad i = 2, \ldots, n \tag{2}$$

Then $\rho \in R^{n-1}$ represents **transverse coordinates**. Note that the equations $\rho_{i-1} = 0, \quad i = 2, \ldots, n$ completely characterize the limit cycle.

It can be shown that in the new coordinates the dynamics has the form:

$$\dot{\theta} = 1 + f_1(\theta, \rho)$$
$$\dot{\rho} = F(\theta)\rho + f_2(\theta, \rho)$$

where

$$f_1(\theta, 0) = 0 \qquad f_2(\theta, 0) = 0 \qquad \frac{\partial f_2(\theta, 0)}{\partial \rho} = 0$$

We can now characterize the stability of the periodic orbit:

Proposition 1 ([11]) *The periodic orbit η is exponentially stable if and only if the (time varying)* ***transverse linearization*** $\frac{d\rho}{d\theta} = F(\theta)\rho$ *is asymptotically (and thus exponentially) stable.*

We will extend this proposition to hybrid periodic orbits in the following section.

2.1 Hybrid systems with impacts

For hybrid periodic orbits, the dynamics can still be decomposed in the same way except that smooth functions become piecewise smooth. As a result, the transverse linearization becomes (time varying) piecewise linear.

In this work we are interested in linear time-varying (LTV) state feedback controllers. It is thus worth examining how the control input enters the equations. Consider the hybrid control system with impact effects:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, & x \notin \Sigma \\ x^+ &= \mathcal{P}(x^-), & x^- \in \Sigma \end{aligned} \tag{3}$$

where u is the control input, Σ is the submanifold on which the impact takes place, x^- is the state just before the impact, x^+ is the state just after the impact, and $\mathcal{P}$ is the state transition function at the impact point that will be discussed later. We will assume that the functions $f(x)$ and $g(x)$ are piecewise smooth, and there is a switching rule that governs the evolution of the hybrid dynamics. We also assume that for $u = 0$ the dynamical system has a periodic orbit $\eta \subset R^n$ with a period T.

Following a similar procedure as above we arrive at the following set of equations for the piecewise smooth part of the hybrid dynamics:

$$\dot{\theta} = 1 + f_1(\theta, \rho) + \Phi_1(\theta, \rho, u)$$
$$\dot{\rho} = F(\theta)\rho + G(\theta)u + f_2(\theta, \rho) + \Phi_2(\theta, \rho, u)$$

where

$$G_i(\theta) = \left(\frac{\partial \psi_{i+1}}{\partial x_j} g_j\right) \circ y(\theta) \text{ and } \Phi_1(\theta, 0, 0) = \Phi_2(\theta, 0, 0) = 0.$$

The functions F, f_1, f_2 are defined as before, and Φ_2 is quadratic in ρ and u.

Since we are ultimately interested in the control of walking robots, the impact phenomenon that we will consider is the plastic impact model for mechanical systems. Let Q be the configuration manifold of a mechanical system evolves, M_1 and M_2 be two submanifolds of Q, and $\Sigma = M_1 \cap M_2$. The state of the system thus corresponds to $x^T = \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix}$. Assume the kinetic energy of the system is given by $T = \dot{q}^T J(q)\dot{q}$. Suppose a trajectory $x : [t_1,t_2] \to Q$ starts at $q_0 \in M_1$ and evolves to $q_1 \in M_2$, with the crossing (impact) from M_1 to M_2 at time $t' \in (t_1,t_2)$. Assume that the two manifolds are described by $M_i = \mu_i^{-1}(0)$. That is, we assume M_i corresponds to the zero set of a function $\mu_i : Q \to \mathbb{R}^{n_i}$. Note that this implies that $\dim M_i = n - n_i$, where $n = \dim(Q)$.

Define the inner product between two tangent vectors of Q at $q \in Q$ by $\langle v_1, v_2 \rangle = v_1^T J(q) v_2$. Let λ_i, $i = 1,\ldots,n_2$ denote a set of orthonormal vectors that are perpendicular to M_2. Then the velocity of the system after the impact can be expressed as $\dot{q}^+ = \dot{q}^- - \sum_{i=1}^{n_2} \langle \dot{q}^-, \lambda_i \rangle \lambda_i$. This equation is linear in the velocities and can be written as $\dot{q}^+ = P\dot{q}^-$ where P is the projection matrix to the tangent space of M_2 at the point of impact.

The overall state impact model can be thus written as:

$$x^+ = \begin{bmatrix} q^+ \\ \dot{q}^+ \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & P \end{bmatrix} \begin{bmatrix} q^- \\ \dot{q}^- \end{bmatrix} = \mathcal{P}x^-. \qquad (4)$$

Now let's consider the impact effects in the transverse dynamics. Assume the impact takes place at $\theta = \theta_0$. Let $\rho_i^+ = \psi_{i+1}^+$ denote the transverse coordinates just before the impact and $\rho_i^- = \psi_{i+1}^-$ denote the transverse coordinates just after the impact. It is clear that we have the following relationship:

$$\rho_i^+ = \psi_{i+1}^+(x^+) = \psi_{i+1}^+(\mathcal{P}x^-) = \psi_{i+1}^+\left(\mathcal{P}\psi^{-1}(\theta_0, \rho^-)\right) \quad (5)$$

Linearizing this equation yields

$$\rho^+ = A_{imp}(\theta_0)\rho^- + B_{imp}(\theta_0, \rho^-).$$

Combining the piecewise smooth part and the impact model altogether yields the overall dynamics in the new coordinates:

$$\begin{aligned} \dot{\theta} &= 1 + f_1(\theta,\rho) + \Phi_1(\theta,\rho,u) && (6) \\ \dot{\rho} &= F(\theta)\rho + G(\theta)u + f_2(\theta,\rho) + \Phi_2(\theta,\rho,u), && (\theta,\rho) \notin \Sigma \\ \rho^+ &= A_{imp}\rho^- + B_{imp}(\theta,\rho^-), && (\theta,\rho) \in \Sigma \end{aligned}$$

Clearly, the procedure can be easily generalized if the system undergoes more than one impact.

Now consider the time varying linear control law $u = \Gamma(\theta)\rho$. We have the following corollary of Proposition 1.

Corollary 1 *Given a linear time-varying control law, a periodic orbit η is exponentially stabilizable if and only if the transversal dynamics*

$$\begin{aligned} \frac{d\rho}{d\theta} &= F(\theta)\rho + G(\theta)u, && (\theta,\rho) \notin \Sigma \\ \rho^+ &= A_{imp}\rho^- + B_{imp}, && (\theta,\rho)^- \in \Sigma \qquad (7) \end{aligned}$$

is asymptotically stabilizable.

The corollary thus ensures that the LTV controller $\Gamma(\theta)$ designed for the transversal dynamics **also stabilizes the original (nonlinear) system to the limit cycle**.

Proposition 1 and Corollary 1 are the principal motivation for using the described dynamics decomposition for control of walking robots. In robot walking, we are typically much more interested in stabilizing the robot, having it move with the right velocity is a secondary goal. The above decomposition provides the decoupling of the two goals and enables us to study the stability of walking separately from the control of the velocity of the robot along the limit cycle. In the approaches that use tracking [4, 5, 6], the two tasks are coupled.

2.2 Discretization

There are no general controller design methods for LTV systems in continuous time. In order to obtain a computational procedure for controller synthesis we thus discretize the transverse dynamics given in Eq. (6). Suppose the periodic orbit has period T. Discretize $[0,T]$ with a grid $0 = \theta_1 \leq \theta_2 \leq \ldots \leq \theta_{N+1} = T$. Let $\theta_p = \theta_{p-1} = \theta p + 1$ be the point of impact on the limit cycle. We thus obtain the following discrete LTV system:

$$\begin{aligned} \rho_{k+1} &= A_k\rho_k + B_k u_k, && k \neq p \\ \rho_{k+1} &= A_{imp}(\theta_k)\rho^k, && k = p \qquad (8) \end{aligned}$$

where A_k and B_k for $k \neq p$ can be computed from $F(\theta)$ and $G(\theta)$ in Eq. (6) using any of the standard methods. Since the transverse dynamics is periodic, we also have $A_k = A_{k+N}$ and $B_k = B_{k+N}$.

Note that after the discretization, the fact that $F(\theta)$, $G(\theta)$ were only piecewise smooth is irrelevant. The impact effects also naturally become an integrated part of the discrete system. The discretization thus naturally allows us to take into account the hybrid nature of the system as well as impacts.

3 Robust controller synthesis

Once the system has been discretized we can design a controller for it. But in order to guarantee that this controller also stabilizes the continuous time system we clearly need to design a controller that is sufficiently robust to disturbances and unmodeled dynamics.

In order to derive a controller synthesis method it is instructive to study the general discrete time robust control problem. Consider the system:

$$\begin{aligned} \rho_{k+1} &= A_k\rho_k + B_k u_k + \mathcal{B}_k\omega_k \\ z_k &= C_k\rho_k + D_k\omega_k \qquad (9) \end{aligned}$$

where ω is a disturbance and z is the output of the system. In our case, the matrix $\mathcal{B}_k$ can be obtained from the system (6) in an analogous way as B_k, while C_k and D_k can be computed once the output for (6) is defined. We would like to design a static robust controller $u_k = \Gamma_k \rho_k$. In this way, the system (9) takes the form:

$$\begin{aligned} \rho_{k+1} &= \mathcal{A}_k \rho_k + \mathcal{B}_k \omega_k \\ z_k &= C_k \rho_k + D_k \omega_k \end{aligned} \tag{10}$$

where $\mathcal{A} = A_k - B_k \Gamma_k$.

The following proposition provides conditions for robust stability of the system (10).

Proposition 2 ([12]) *A LTV system $\{\mathcal{A}, \mathcal{B}, C, D\}$ is robustly stable if and only if*

$$1 \notin \text{spec}(Z\mathcal{A}) \text{ and } \|C(I - Z\mathcal{A})^{-1} Z\mathcal{B} + D\| < 1 \tag{11}$$

where Z is a shift operator defined by $Z\rho_k = \rho_{k+1}$.

It can be shown that these conditions are equivalent to a set of linear matrix inequalities (LMIs) on the set of matrices defining the system (10).

Proposition 3 ([12]) *The conditions from Proposition 2 are satisfied if and only if there exists a sequence of matrices $X_k > 0$ bounded above and below that satisfies the following set of LMIs for all indices k:*

$$\begin{bmatrix} \mathcal{A}_k & \mathcal{B}_k \\ C_k & D_k \end{bmatrix}^T \begin{bmatrix} X_{k+1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \mathcal{A}_k & \mathcal{B}_k \\ C_k & D_k \end{bmatrix} - \begin{bmatrix} X_k & 0 \\ 0 & I \end{bmatrix} < 0 \tag{12}$$

This result appears not to be directly applicable to the controller synthesis since the above system of LMIs is infinite dimensional. However, we are interested in stabilizing the transverse dynamics which is periodic. For periodic systems, the system (12) becomes finite and can be thus used for computation.

What prevents us from directly using equations (12) for controller synthesis is that the matrices $\mathcal{A}_k$ are **closed loop** matrices and they depend linearly on the unknown gain matrices Γ_k. The system (12) is therefore quadratic in the unknowns. However, one of the main contributions of this work are the following four propositions showing that Eq. (12) can be transformed into a set of LMIs. We omit the proofs in the interest of space.

Proposition 4 *Suppose that the period of the discrete LTV system is N, then there exists an admissible static robust LTV controller for the system if and only if there exists a finite sequence of matrices $X_k > 0$ such that the following LMIs are satisfied:*

(a)

$$\begin{bmatrix} -I & -\mathcal{B}_k^T & -D_k^T \\ -\mathcal{B}_k & -X_{k+1} & 0 \\ -D_k & 0 & -I \end{bmatrix} < 0 \tag{13}$$

(b)

$$\begin{bmatrix} -X_k & 0 & -X_k A_k^T V_k & -X_k C_k^T \\ 0 & -I & -\mathcal{B}_k^T V_k & -D_k^T \\ -V_k^T A_k X_k & -V_k^T \mathcal{B}_k & -V_k^T X_{K+1} V_k & 0 \\ -C_k X_k & -D_k & 0 & -I \end{bmatrix} < 0 \tag{14}$$

for all indices $k = 1, 2, \ldots, N$, where $\operatorname{Im} V_k = \operatorname{Ker} B_k^T$ and $X_{N+1} = X_1$.

The proposition gives the necessary and sufficient condition for the existence of a controller. The following corollary shows how to actually compute it.

Corollary 2 *If there is a solution to the LMI systems (13) and (14), there exists a sequence of matrices Γ_k such that the following LMIs are satisfied:*

$$H_k + P_k^T \Gamma_k Q_k + Q_k^T \Gamma_k^T P_k < 0 \tag{15}$$

where

$$H_k = \begin{bmatrix} -X_k & 0 & -X_k A_k^T & -X_k C_k^T \\ 0 & -I & -\mathcal{B}_k^T & -D_k^T \\ -A_k X_k & -\mathcal{B}_k & -X_{k+1} & 0 \\ -C_k X_k & -D_k & 0 & -I \end{bmatrix}$$

$$P_k = \begin{bmatrix} 0 & 0 & B_k^T & 0 \end{bmatrix}$$

$$Q_k = \begin{bmatrix} -X_k & 0 & 0 & 0 \end{bmatrix}$$

for all indices $k = 1, 2, \ldots, N$. The sequence of matrices $\Gamma_k > 0$ corresponds to the gains for the admissible static robust LTV controller.

Since there exist efficient tools for solving LMIs, this corollary directly provides the computational method for robust controller synthesis. The dimension of the system of LMIs is proportional to nN, where n is the dimension of the transversal dynamics and N is the number of points at which the limit cycle is discretized. The procedure therefore **scales linearly** with the dimension of the system.

It is worth pointing out that the described controller synthesis procedure is fundamentally different from gain scheduling. There, the individual controllers are designed independently and the rate at which the dynamics of the system changes can adversely affect the stability. The proposed method instead directly guarantees the stability of the system since its entire evolution along the limit cycle is considered during the controller design.

4 Nominal input and tangential dynamics control

In general, a walking robot does not naturally have the limit cycle we want to follow. However, control inputs can be used to force the system to have the desired limit cycle. Given that the limit cycle is characterized by equations $\rho_{i-1} = 0, \ i = 2, \ldots, n$, we can compute the nominal dynamics of the system:

$$\dot{x} = f_d(x) \tag{16}$$

If the limit cycle is properly designed, we can thus solve for the open-loop nominal input u_0 from the following equation:

$$f(x) + g(x)u_0 = f_d(x) \qquad (17)$$

Also, since the tangential dynamics is decoupled from the transversal dynamics we can design the controller for the tangential dynamics directly from Eq. (6). Unfortunately, it is typically hard to obtain this equation in the analytic form. An alternative is to take advantage of the properties of the limit cycle. We often use one of the joints to characterize the tangential motion in limit cycle design. We can thus use one of the control inputs to stabilize this joint to its own limit cycle and use the rest of the inputs to control the transverse dynamics. This is equivalent to forcing the controller Γ_k to have a block diagonal form instead of a fully rectangular one and can be easily done when setting up the LMIs.

5 Implementation of the controller

Controllers designed in section 3 must be applied to the original control system (3). In order to apply the controller we therefore need to determine k so that the proper controller Γ_k is applied. It is easy to see that we must choose $k = \left[\frac{\theta N}{T}\right]$ where $[x]$ means the integer that is closest to x. However, in order to determine θ we must solve online the following minimization problem: $\theta = \arg\min_{\theta \in [0,T]} \|y(\theta) - x\|$.

Since the periodic orbit has been uniformly discretized there is an alternative way to choose the right controller. We compute the distances between the current state x and the points at which the limit cycle has been discretized. The index of the point on the limit cycle closest to the current state corresponds to the correct index k for the controller Γ_k. Note that this is a constant time (for fixed N) computation. In this way it is also easy to enforce conditions such as the requirement that a particular controller can be only chosen when the system is in the appropriate walking phase.

6 Example

In this section we will use a simple planar biped robot to illustrate our approach. The schematic of the robot is shown in Figure 1. Its trunk is simplified to a point mass whose orientation is not considered. It is assumed that the walking cycle takes place on a level surface. The robot is fully actuated. We are considering full dynamics so the dimension of the state space is 8. We emphasize that although such a simple model is used here, our approach applies equally well to under-actuated walking robots with more complex limit cycles, where other approaches may be extremely difficult or even infeasible.

Each time the robot lifts the leg from the ground or puts it on the ground the system dynamics changes. It can be therefore modeled as a hybrid system with impact effects. We assume that the walking cycle consists of successive phases of single support, and the transition from one leg to the other is instantaneous. The assumptions are common in the robotics literature [9, 15].

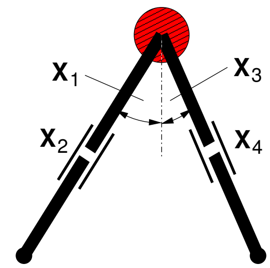

Figure 1: Configuration of the biped robot.

For simplicity a limit cycle was designed so that the trunk moves on a horizontal line. A more natural trajectory could be easily substituted. The tangential coordinate depends on the dynamic regime and was characterized by the angle of the forward (or stance) leg from the vertical line. It can be easily seen that this angle strictly monotonically decreases. Referring to Figure 1, assume that the tangential coordinate is x_3. The functions $\psi_i,\ i = 2,\ldots,8$ that define the limit cycle are all functions of x_3 and can be easily computed from the geometry. The nominal input u_0 that results in the desired limit cycle can be then computed to decouple x_3 and $\dot{x}_3$ from the other variables, as described in section 4.

The limit cycle was discretized uniformly with $N = 100$ points. The computation of the controller given by Eq. (15) was performed in Matlab using the LMI Control toolbox. The computation is performed off-line, but the controller is then applied online according to the procedure described in Section 5.

Figure 2 shows the evolution of the system for an initial condition that is not on the limit cycle. The states x_1 and x_3 correspond to the variable θ in the corresponding regions. Note that the spikes in the transversal dynamics are caused by the impact effects as well as the changes in the system dynamics, since transverse coordinates have different definition in each dynamic regime. It can be seen that although the initial condition is not on the periodic orbit, the system is quickly attracted to it and the robot maintains a stable walking gait. The simulation therefore indicates that the periodic orbit is indeed stabilized.

7 Conclusions and future research

The paper describes a computational method for stabilization of periodic orbits for hybrid systems with impacts. The method is applied to synthesis of controllers for a bipedal walking robot. Two main characteristics of the method are that the dynamics of the system along the periodic orbit is partitioned into tangential and transversal components, and that LMI-based robust control techniques are used to perform the controller design on the discretized system.

The method has several main advantages: (a) it scales up linearly with the dimension of the system; (b) all the steps can be performed automatically apart from choosing the periodic orbit; and (c) the hybrid nature of the system can be easily accommodated since the controller design is performed

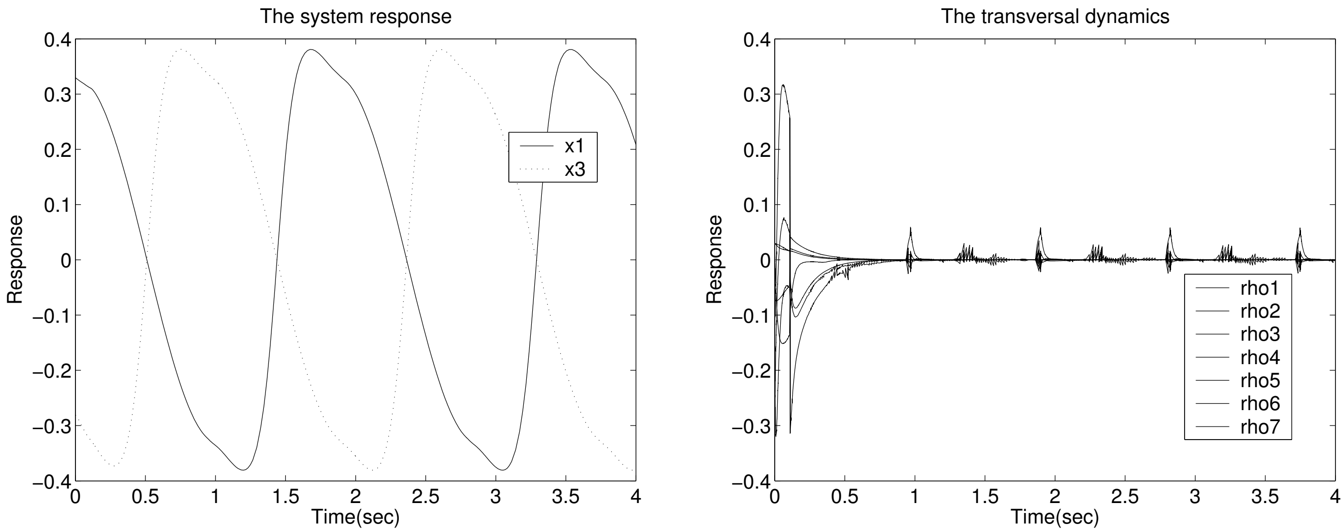

Figure 2: Time response of the system.

on the discretized system.

There are several issues that remain to be addressed. First, we plan to apply the method to a more realistic example and to perform experiments on a hardware platform that is currently under development. Second, some numerical issues need to be resolved in order to apply the approach to more complex examples and under-actuated robots. Third, we need to define walking primitives so that different kinds of gaits can be realized and adaptation of trajectory can be accomplished to make this approach more flexible. We believe that this work can be easily extended to account for these phenomena. We also need to estimate the errors introduced in the discretization of the dynamics. This would provide an estimate on the number of points at which the periodic orbit needs to be discretized. And finally, we would like to derive sufficient conditions that would guarantee the existence of stabilizing controllers for a given periodic orbit.

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