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Optimal Trajectories and Force Distribution for Cooperating Arms

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Abstract

The optimization of trajectories and actuator torques for a dual arm manipulation system is considered. Given the starting and final configurations, we find the trajectories that minimize (a) the integral of the norm of the vector of derivatives of the actuator forces; and (b) the integral of the norm of the actuator forces. In this way both kinematic and actuator redundancy are resolved. The optimization problem reduces to solving a two-point boundary value problem for coupled, nonlinear differential equations. The effect of different parameters such as preload and inertia are investigated and the results are compared with those obtained using other well-known cost functions.

1 Introduction

We address the problem of planning optimal motions with coordinated robot arms, cooperatively manipulating an object, where the task is to transport the object from one configuration to another. Because there are many viable trajectories, it is meaningful to look for the best trajectory. When dealing with coordinated robot arms, the problem of determining the joint torques for a desired trajectory is also underconstrained. We are thus interested in an optimal trajectory and an optimal load-sharing scheme.

Previous work in this direction has focused on minimum-time trajectories for a single unconstrained robot arm. Minimum-time control for a single robot arm has been studied in [1, 2]. In these early works, the optimization problem is simplified by the fact that a prespecified path confines the motion of the system to only one degree of freedom expressed by the path parameter. More recently, Shiller and Dubowsky developed a method for computing the time-optimal motions of robotic manipulators that considers the manipulator dynamics, actuator constraints, joint limits, and obstacles [3]. While the optimized trajectory enables the task to be performed in minimum time, it requires one or more actuators to switch between their maximum and minimum efforts instantaneously. Because actuators have finite response times, the resulting trajectory may not follow the desired trajectory.

Human reaching has been studied extensively. The question that naturally arises is which control principle humans use in such motions. Flash and Hogan [4, 5] suggested that the central nervous system uses an optimality criterion to calculate the trajectory along which to move. In medium speed, large amplitude, unconstrained planar motions for a single arm, the integral of the jerk along the trajectory is minimized. The *minimum-jerk* solution, as it is known in the literature, depends only on the kinematics of the task and is independent of the physical structure and the dynamics of the arm. The advantage of the scheme, namely its simplicity, is also a disadvantage: The trajectory is independent of the mechanical system executing the task and the scheme does not incorporate the dynamic constraints of the system.

Kawato et al. [6] proposed an alternative cost function for trajectory generation. This function is the integral of the norm of the vector of derivatives of the joint-torques along the trajectory, hence the name *minimum torque-change criterion*. Although computationally more intensive and more complex than the minimum-jerk hypothesis, it incorporates the dynamics of the system. It is more attractive than the minimum-time criterion in the sense that it favors smooth changes in torques as opposed to a bang-bang switching strategy.

Optimal trajectories for coordinated manipulation or reaching with two arms by humans have received less attention. For robot systems, Shin and Zheng proposed an algorithm for finding minimum-time trajectory for a dual arm system where each arm moves along its own path prespecified by the user [7]. Chen studied the structure of time-optimal control law for multiple robot arms cooperatively moving a common object along a specified path with joint torque constraints [8]. Bien and Lee proposed a minimum-time trajectory planning method for two arms considering joint torque and velocity constraints [9].

In this paper, we study the optimization of two-arm manipulation tasks using three different criteria:

- The minimum torque-change criterion, in which the cost function is the integral of the norm of the vector of derivatives of the actuator forces;
- The minimum torque criterion, which minimizes the integral of the norm of the actuator forces;

- The minimum-jerk criterion, which minimizes the integral of the Cartesian jerk, to find the optimal trajectory, and the pseudo-inverse solution to resolve the kinematic and actuator redundancy.

In the first two cases, the optimization problem reduces to solving a two-point boundary value problem with coupled, nonlinear differential equations. A finite-difference method is used to solve the boundary value problem. The optimization yields the optimal path and exploits the available kinematic and actuator redundancy to yield optimal joint trajectories and actuator forces/torques. In contrast, last criterion, which is typical of schemes generally seen in the robotics literature, yields analytical expressions for the trajectory and locally-optimal solutions for joint rates and actuator forces. A simulation of two planar cooperating arms is used to demonstrate the central ideas, the application of the method and to compare the criteria.

2 Dynamic modeling

We consider two manipulators holding an object. Each manipulator has n links and n actuated degrees of freedom. We further assume that the interaction between the two manipulators through the object can be modeled as a k degree-of-freedom kinematic pair, and this is true regardless of the contact forces. In other words, there are no constraints on the forces exerted on the object. Equivalently, we can say that the interaction can be described by $m = c - k$ constraint equations, where $c = 3$ for the planar motion and $c = 6$ for spatial motion. If the constraints are holonomic we can express them as:

$$g(\theta) = 0 \qquad (1)$$

where g is a $m \times 1$ vector function and θ a $2n \times 1$ vector of joint variables. Further, we assume that the object is a point mass. This is a reasonable assumption if the size of the object is small in comparison to the link lengths.

Dynamic equations of the closed chain formed by the two manipulators and the object can be derived using Lagrange multipliers:

$$\begin{aligned} I_1(\theta_1)\ddot{\theta}_1 + C_1(\theta_1, \dot{\theta}_1) + G_1(\theta_1) &= \tau_1 + \Gamma_1^T \lambda \\ I_2(\theta_2)\ddot{\theta}_2 + C_2(\theta_2, \dot{\theta}_2) + G_2(\theta_2) &= \tau_2 + \Gamma_2^T \lambda \end{aligned} \qquad (2)$$

where θ_i is an $n \times 1$ vector of the joint coordinates of the ith $(i = 1, 2)$ manipulator, $I_i(\theta)$ is an $n \times n$ inertia matrix, $C_i(\theta_i, \dot{\theta}_i)$ is an $n \times 1$ vector of nonlinear terms (Coriolis and centrifugal forces), G_i is an $n \times 1$ vector of gravity terms, τ_i is an $n \times 1$ vector of the joint torques, $\Gamma_i = \frac{\partial g}{\partial \theta_i}$ is an $m \times n$ matrix of the derivatives of the constraint equations with respect to the joint coordinates θ_i, and λ is an $m \times 1$ vector of Lagrange multipliers. Note that the inertia of the object can be incorporated into the inertia matrix of one of the manipulators. In this paper we lump it with the inertia of the distal link of manipulator 1.

The closed kinematic chain has $p = 2n - m$ degrees of freedom. If the task space has dimension r $p \geq r$ must hold. In general there will be $p - r$ surplus degrees of freedom. In other words, the system is kinematically redundant. Since all the joints are actuated, we also have $2n - p$ redundant actuators. The system is therefore statically redundant and possesses actuator redundancy. In summary, for the task of positioning an object with two arms, the system possesses actuator redundancy $(2n - p > 0)$, and in addition it may be kinematically redundant (if $p > r$).

3 Optimal control

Only an initial and a final (task space) configuration of the closed chain formed by the two manipulators and the object are prescribed for the manipulation task. To perform the task we have to choose between the infinitely many trajectories that connect the two configurations. If the mechanism is kinematically redundant then the trajectory of the object does not uniquely determine the joint angles. And even if the configuration of the linkage (joint angles) is known at every point on the trajectory, the joint torques cannot be uniquely determined if actuators are redundant. Therefore, there will be in general three levels of indeterminacy in manipulation problem:

1. trajectory has to be chosen,
2. kinematic redundancy has to be resolved,
3. actuator redundancy has to be be resolved.

In this section we address the global optimization based on the minimum torque-change criterion. The formulation for the minimum torque criterion is similar. The formulation for minimum-jerk criterion is discussed in [5, 10].

By eliminating Lagrange multipliers from equations (2), we are left with $2n - m$ differential equations. In addition m variables can be eliminated from these equations using constraint equations (1) to obtain a set of $2n - m$ differential equations relating $2n - m$ joint accelerations and $2n$ joint torques. Furthermore, r initial and r final conditions on the remaining $2n - m$ joint variables are specified by the manipulation task. A necessary condition to obtain a unique optimal solution for the joint variables and the torques is that the cost function depends on each joint torque or its derivative. The missing boundary conditions are obtained from the optimality conditions in the form of the boundary conditions in the adjoint variables [11].

Because of the space restrictions we only consider the minimum torque-change criterion and the less general case in which the system is not kinematically redundant. That is, $2n - m = r$. The cost function, J, is given by

$$J = \frac{1}{2}\int_{t_0}^{t_f} \dot{\tau}^T \dot{\tau}\, dt \tag{3}$$

where $\tau = \left[\tau_1^T \tau_2^T\right]$ and $\dot{\tau}$ is the time derivative of τ. The solution of (3) must also satisfy $2n - m$ second order differential equations obtained after eliminating dependent variables from dynamic equations (2).

The methods for solving constrained optimization problems are well established [11]. In order to use the standard techniques the set of dynamic equations must be rewritten in the state space. First, a $2n \times 1$ vector u of control variables is introduced:

$$u = \dot{\tau}. \tag{4}$$

A $(4n - 2m) \times 1$ state vector formed by independent joint angles and corresponding joint velocities is introduced:

$$x_p = \begin{bmatrix} \theta \\ \omega \end{bmatrix}, \tag{5}$$

where $\omega = \dot{\theta}$. To include equations (4) into the state space the state vector x_p is extended [12] so that it includes the torques. The complete state of the system is described by the vector:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \theta \\ \omega \\ \tau \end{bmatrix}, \tag{6}$$

where τ is a $2n \times 1$ vector of the joint torques of the left and right arms. The dimension of the state space is therefore $6n - 2m$. The state equations can now be expressed in the form:

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{\tau} \end{bmatrix} = \begin{bmatrix} x_2 \\ A(x_1, x_2) + B(x_1)x_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} u \tag{7}$$

where u as defined in (4) represents the input to the system. Symbolically, the equation (7) can be written as:

$$\dot{x} = f(x, u) \tag{8}$$

We are now able to restate our original optimal control problem in the form:

$$\min_u \frac{1}{2}\int_{t_0}^{t_f} u^T u\, dt \tag{9}$$

subject to equation (8) and the boundary conditions

$$x_A(t_0) = x_0, \qquad x_B(t_f) = x_f, \tag{10}$$

where x_A and x_B are $r \times 1$ subsets of the vector of the joint variables θ.

The derivation of the solution of the optimal control problem (9) is taken from Sections 2.3 and 2.4 in [11]. First we introduce a Hamiltonian

$$H(x, u, \nu) = \frac{1}{2}u^T u + \nu^T f(x, u), \tag{11}$$

where ν is known as a vector of influence functions [11] or adjoint variables. The optimal solution, if it exists, must satisfy the following system of $12n - 4m$ first order differential equations:

$$\begin{aligned} \dot{x} &= f(x, u), \\ \dot{\nu} &= -\left(\frac{\partial f}{\partial x}\right)^T \nu, \end{aligned} \tag{12}$$

subject to the boundary conditions

$$\begin{aligned} x_A(t_0) &= x_0, & \nu_{A'}(t_0) &= 0, \\ x_B(t_f) &= x_f, & \nu_{B'}(t_F) &= 0. \end{aligned} \tag{13}$$

In equation (13) $\nu_{A'}$ and $\nu_{B'}$ are those adjoint variables whose corresponding state-space variables are not prescribed at $t = t_0$ or $t = t_f$, respectively. The boundary conditions would determine the solution of the system of $12n - 4m$ first order differential equations (12) if the input vector u were known. The input vector is given by the optimality condition:

$$\frac{\partial H}{\partial u} = 0. \tag{14}$$

Substituting from (11) we get,

$$u = -\left(\frac{\partial f}{\partial u}\right)^T \nu \tag{15}$$

The optimal solution is thus obtained by solving two-point boundary value problem (12), (13) where u is given by (15).

4 Two Planar Cooperating Arms

We study the case of two 2-R manipulators in the horizontal plane holding a small object as shown in Figure 1. Here each manipulator has two actuators ($n = 2$) and the task space (the position of the object) is two-dimensional ($r = 2$). Since the object is assumed to be small (compared to the smallest link length), the interaction between the two manipulators can be modeled as a revolute joint ($k = 1$). Thus, the number of constraints, m, is equal to two and the system has two degrees of freedom ($p = 2$). In fact, the kinematic chain is a five-bar linkage whose mobility is known to be two. The four joint angles θ_1 to θ_4 can be obtained if the position of the object is specified. Hence, the linkage does not exhibit any kinematic redundancy ($p - r = 0$). However, because the number of actuated joints is four, the system possesses actuator redundancy ($2n-p=2$).

The dynamics of the system is given by equation (2). The 2×2 matrix, Γ_1, is the Jacobian for manipulator 1 while Γ_2 is the Jacobian for the second manipulator multiplied by a negative sign. The other matrices and vectors are as defined before. Because the manipulators are moving in the horizontal plane, the vectors G_i vanish.

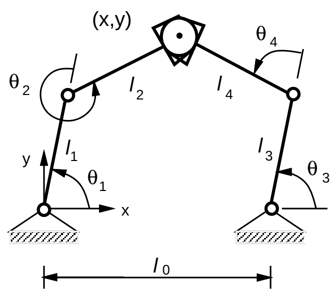

Figure 1: Two planar two-link arms holding an object.

The system of equations (2) can be simplified by eliminating the Lagrange multipliers:

$$
\begin{gathered}
I_1(\theta_1)\ddot{\theta}_1 + C_1(\theta_1,\dot{\theta}_1) = \\
\tau_1 + \Gamma_1^T(\Gamma_2^T)^{-1}(\tau_2 - I_2(\theta_2)\ddot{\theta}_2 - C_2(\theta_2,\dot{\theta}_2))
\end{gathered} \tag{16}
$$

The expressions for I_i, C_i, and Γ_i for a two-link manipulator can be found in a number of standard robotic textbooks (see for example, [13]).

A few general observations about the dependence of the optimal solutions on the duration of the task can be made that are true for all three investigated criteria. In this study the duration of the motion is fixed. However, the solution scales with the duration of motion: if $\theta(t)$ is a solution of the boundary value problem for $t_f = T_1$, then $\theta(tT_1/T_2)$ is a solution for the same problem with $t_f = T_2$. The other variables (velocities, torques and adjoint variables) are also multiplied by the appropriate factor. Also, the time can be reversed: if $\theta(t)$ is a solution for the boundary values $x(t_0) = x_0$ and $x(t_f) = x_f$, then the solution for the boundary values $x(t_0) = x_f$ and $x(t_f) = x_0$ is given by $\theta(t_f - t)$. The later is only true if the boundary conditions in the other variables (velocities and torques or possibly adjoint variables) are the same at both endpoints.

4.1 Numerical method

To obtain the optimal solution with the minimum torque-change criterion, we must solve the problem described by equations (9). This in turn translates to the boundary value problem with the system of 16 first order differential equations (12) with the boundary conditions (13). Because the equations are nonlinear analytical solutions cannot be found and therefore a numerical method is used. If the minimum torque criterion is used a similar approach can be pursued but we only have 8 first order differential equations and the system is much simpler. Because the minimum torque-change criterion results in the most complex optimization problem, only this criterion is discussed in detail.

Two numerical methods were explored: (a) shooting method; (b) finite difference method. In the shooting method an estimate for the unspecified initial values is iteratively refined until all the specified boundary conditions are satisfied. However, this method is very sensitive to the initial guess for the values of the adjoint variables ν and failed to converge unless the guess was sufficiently close to the correct solution. In the finite difference method the time interval is discretized and the derivatives approximated with finite differences. An initial trajectory is assumed. Newton's method is used to find the corrections for the (discretized) state variables at the mesh points. This requires derivatives of the finite-difference equations with respect to these variables. Because the system equations are too complicated the derivatives are estimated numerically. This method is quite robust with respect to the choice of the parameters (physical properties of the two manipulators, boundary conditions, initial values for the numerical method). For a discretization in time with 100 to 1000 points, it typically yields a solution in less then 10 iterations.

In contrast to the other two criteria, the minimum jerk criterion leads to an analytical solution for the trajectory [4]. The path is a straight line and the Cartesian coordinates vary as a fifth degree polynomial function of time. However, the torques are not determined by this approach. It is possible to use a pointwise minimum torque criterion that would minimize the vector of joint torques at each point on the optimal trajectory. This is easily done by employing a pseudo-inverse of the 2×4 matrix $\Gamma = [I \quad \Gamma_1^T(\Gamma_2^T)^{-1}]$:

$$
\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \Gamma^+ \cdot \{I_1\ddot{\theta}_1 + C_1 + \Gamma_1^T(\Gamma_2^T)^{-1}(\tau_2 - I_2\ddot{\theta}_2 - C_2)\} \tag{17}
$$

4.2 Simulation Results

The physical dimensions for each arm are typical of the human arm [6]. In each manipulator the length of the proximal link is $0.27m$ and its mass $0.9kg$. The distal link is $0.33m$ long with a mass of $1.1kg$. The distance between the bases of the two arms is $0.45m$. The coordinate system is located at the base of the left arm (the base of the right arm is at $(0.45m, 0m)$). For the results presented in this section, the starting

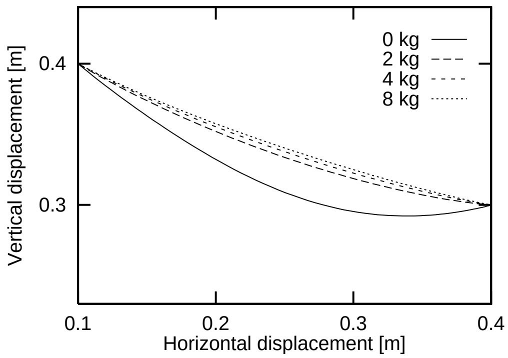

Figure 2: Trajectories for different object mass.

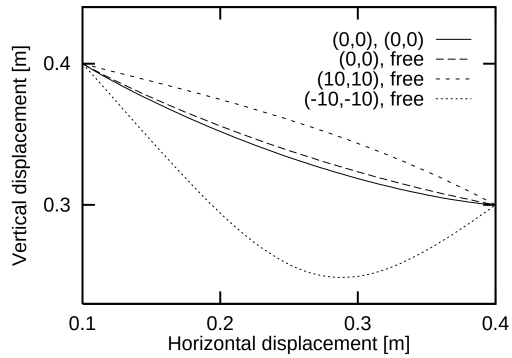

Figure 3: Trajectories for different preload forces.

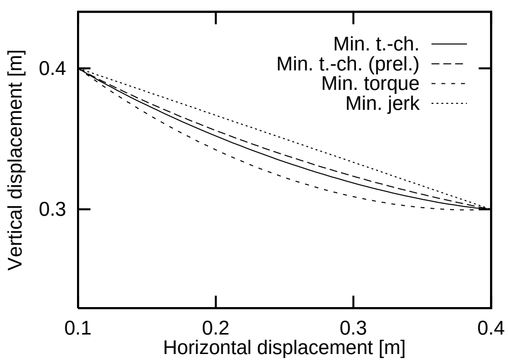

Figure 4: Trajectories for different cost functions.

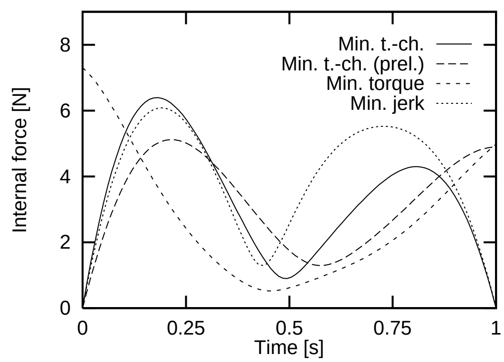

Figure 5: Magnitude of the internal force for different cost functions.

position is at point $(0.1m, 0.4m)$ and the end position is at $(0.4m, 0.3m)$, the starting and end velocities are zero, and the time duration is $1.0s$.

First, the effect of the mass of the object on the trajectories is investigated. The results for the minimum torque-change criterion with the object mass of $0kg$, $2kg$, $4kg$ and $8kg$ are shown in Figure 2. When the mass of the object is small compared to the masses of the links, the trajectory is in general curved. As the mass increases the trajectories approach the straight line joining the starting and end positions. However, even in the limit when the masses of the links become negligible compared to the mass of the object, the results from the minimization of the derivative of the joint torques are different from those obtained from the minimization of the Cartesian jerk which yields a straight line as the optimal solution [4].

Note that the initial and final values for the torques were specified to be 0 for the plots on Figure 2. However, a two-arm grasp can be preloaded with an internal force that performs only isometric work prior to the beginning of motion. Here, the internal force, (F_x, F_y), is the vector difference of the forces applied on the object by the left arm and the forces applied by the right arm. A non-zero internal force at the starting configuration results in non-zero initial torques. In Figure 3 the effect of the preloading on the trajectories is shown for $(F_x, F_y) = (0N, 0N)$, $(10N, 10N)$, and $(-10N, -10N)$ for an object mass of $2kg$. The internal forces (the preload torques) at the end of the motion are *not* prescribed (therefore the corresponding adjoint variables are prescribed to be 0). This is denoted by “free” in Figure 3. The trajectory for zero initial and final torques is denoted by “(0,0), (0,0)” in the figure. The figure shows that preload forces greatly affect the shape of the trajectory. It is well known that in a system with agonist/antagonist actuation the preload is directly related to the stiffness of the system. Changing the apparent initial stiffness thus alters the trajectory that the system follows.

The solutions for three different criteria are compared in Figures 4 and 5. The minimum torque-change criterion results in different curves when the initial and final torques are prescribed to be zero (denoted by “Min. t.-ch.”), and when only the initial torque is zero while the final torque is not prescribed (denoted by “Min. t.-ch. (prel.)”). The results from the minimum torque criterion are also the results for the minimum energy criterion (it is generally accepted that the energy spent by DC electric motors is proportional to the square of the motor torques). The joint torques with the minimum (Cartesian) jerk criterion are calculated using equation (17). For all these plots the object mass is $2kg$. The trajectory for the minimum torque solution is the most curved. The minimum torque-change solutions for specified final torques and those for unspecified final torques are progressively straighter. Finally the minimum-jerk solution is a straight line. It is also interesting to compare the magnitude of the internal force during the motion (Figure 5). The figure shows that the minimum torque criterion also gives the lowest average internal forces. Both versions of the minimum torque-change criterion have comparable average internal forces, while the minimum- jerk trajectory with torques given by a pseudo-inverse has the highest average internal force.

5 Discussion

This paper addresses the optimization of dynamic performance of a robot system at three levels. First, it allows the selection of an optimal trajectory. In principle, we can incorporate constraints other than the state equations (such as frictional constraints) on the state variables in the formulation. Second, for kinematically redundant mechanisms it allocates the motion between the different joints. Finally, it automatically optimizes the distribution of forces between the actuators. Although a simple example was analyzed in the paper, the basic ideas and the method are extensible to more complex problems. While the minimum-time control of robot systems has been studied extensively, this is the first attempt to optimize the force distribution (and trajectory) in a robot system with closed-chains.

The algorithms proposed in the paper are well-suited to off-line trajectory planning of cooperative manipulation tasks. If the optimization is implemented using a finite difference approach the technique is fast and robust. On a Sun Sparc 10 workstation it tipically takes around 3 seconds to compute the solution.

There is considerable interest in studying the fundamental problems underlying the coordination of arms in human manipulation tasks. While considerable work has been done in this area in single-arm reaching tasks, to our knowledge this is the first paper to focus on cooperative, two-arm manipulation tasks. Different cost functions and their effects on system performance were studied in this paper. This study complements the ongoing experimental study of human manipulation [14]. Comparisons between the simulation predictions and experimental observations will be reported in future publications.

Acknowledgment

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References

[1] J. Bobrow, S. Dubowsky, and J. Gibson, "Time-optimal control of robotic manipulators along specified paths," *Int. J. Robotic Research*, vol. 4, no. 3, 1985.

[2] K. G. Shin and N. D. McKay, "Minimum-time control of robotic manipulators with geometric constraints," *IEEE Trans. Automatic Control*, vol. AC-30, no. 6, 1985.

[3] Z. Shiller and S. Dubowsky, "On computing the global time-optimal motions of robotic manipulators in the presence of obstacles," *IEEE Trans. Robotics and Automation*, vol. 7, no. 6, 1991.

[4] N. Hogan, "An organizing principle for a class of voluntary movements," *J. Neuroscience*, vol. 4, no. 11, 1984.

[5] T. Flash and N. Hogan, "The coordination of arm movements: An experimentally confirmed mathematical model," *J. Neuroscience*, vol. 5, no. 7, 1985.

[6] Y. Uno, M. Kawato, and R. Suzuki, "Formation and control of optimal trajectory in human multijoint arm movement," *Biological Cybernetics*, vol. 61, 1989.

[7] K. G. Shin and Q. Zheng, "Minimum time trajectory planning for dual robot systems," in *Proc. 28th IEEE Conf. Decision and Control*, 1989.

[8] Y. Chen, "Structure of the time-optimal control law for multiple arms handling a common object along specified paths," *IEEE Trans. Automatic Control*, vol. 37, no. 10, 1992.

[9] Z. Bien and J. Lee, "A minimum-time trajectory planning method for two robots," *IEEE Trans. Robotics and Automation*, vol. 8, no. 3, 1992.

[10] Y. Zheng and J. Luh, "Optimal load distribution for two industrial robots handling a single object," in *IEEE Int. Conf. Robotics and Automation*, (Philadelphia, PA), 1988.

[11] A. E. Bryson and Y.-C. Ho, *Applied Optimal Control*. New York: Hemisphere Publishing Co., 1975.

[12] X. Yun and V. Kumar, "An approach to simultaneous control of trajectory and interaction forces in dual-arm configurations," *IEEE Trans. Robotics and Automation*, vol. 7, no. 5, 1991.

[13] H. Asada and J.-J. E. Slotine, *Robot Analysis and Control*. New York: John Wiley & Sons, 1986.

[14] G. Garvin *et al.*, "A study on optimality criteria for two-hand reaching tasks," in *13th Southern Biomed. Eng. Conf.*, (Rockville, MD), 1994.