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Passivity of hybrid systems based on multiple storage functions[*]

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Abstract

Passivity is a powerful paradigm for design of nonlinear control systems. We have previously shown [18] that the basic results from the passivity theory for smooth systems can be extended to hybrid systems if the notion of passivity is suitably modified. In this paper we explore an extension of those results by proposing a definition of passivity for hybrid systems that is based on a more general notion of stability for hybrid systems.

Keywords: hybrid systems, passivity, multiple Lyapunov functions, partial order

1 Introduction

The aim of this paper is to extend the results from [18] where a framework for the passivity analysis of hybrid systems was proposed. That work was motivated by the fact that passivity has not received much attention in the hybrid systems community (see also [17]). The treatment in [18] was based on conditions for stability of hybrid systems proposed in [6]. However, those conditions and subsequently the results in [18] focus on the continuous dynamics of the system, no properties of the discrete dynamics are exploited. A generalized framework for stabilization of nonlinear hybrid systems that exploits the properties of the discrete dynamics was proposed in [19] and is the basis of the approach in this paper.

The paper is organized as follows. We first define our model for a hybrid system. We then briefly review the definition of passivity for continuous systems and show why it is desirable to generalize this notion. Section 3 contains the main results: a definition of passive hybrid systems with partial order (PHSPO) and the relation between this notion of passivity and stability of hybrid systems. We conclude the paper with an example of a haptic display interacting with a virtual environment where the notion of PHSPO can be used to show that the interaction will be stable.

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2 Hybrid system model

Several formal models for hybrid systems have been proposed in the past [5, 7, 8, 15]. Typically, a model is selected according to a problem to be addressed. We will mainly follow the approach in [2].

Intuitively, a hybrid system can be described as a finite set of discrete states, with each discrete state corresponding to a different continuous dynamics. The state of a hybrid system is therefore composed of discrete and continuous components. The evolution of the continuous state can be described by a vector field that is a function of the continuous control. In general it might be possible to force the system to switch from one discrete state to a different discrete state. We will assume that the continuous state does not change during such switches. A selection of a discrete state can be modeled by a set of discrete inputs controlling the evolution of the discrete dynamics. The formal model of a hybrid system can be thus given as:

Definition 1 *A **hybrid system** is a tuple:*

$$HS = (\Xi, \mathcal{M}, \Gamma, \mathcal{U}, \Sigma, \mathcal{F}, \mathcal{H}) \tag{1}$$

where

1. $\Xi \subset \mathbb{Z}$ *is a (finite) set of discrete states.*
2. $\mathcal{M} = \{M_i\}_{i\in\Xi}$ *is a collection of (differentiable, connected) manifolds. For simplicity we assume* $M_i \subseteq \mathbb{R}^n$.
3. $\Gamma \subset \mathbb{Z}$ *is a set of discrete inputs.*
4. $\mathcal{U} \subset \mathbb{R}^m$ *is a set of continuous inputs.*
5. $\mathcal{F} = \{f_i\}_{i\in\Xi}$ *is a set of* ($\mathcal{C}^1$) *controlled vector fields:*

$$\begin{aligned} f_i : \quad & M_i \times \mathcal{U} \rightarrow TM_i \\ & (x,u) \mapsto f_i(x,u) \in T_xM \end{aligned}$$

6. $\Sigma : \Xi \times \mathbb{R}^n \times \Gamma \times \mathcal{U} \rightarrow \Xi$ *is a function describing the discrete evolution of the system.*
7. $\mathcal{H} = \{h_i\}_{i\in\Xi}$ *is a set of* ($\mathcal{C}^1$) *output maps* $h_i : M_i \times \mathcal{U} \rightarrow \mathbb{R}^m$.

The evolution of a hybrid system can be described as follows. The system evolves on M_i following the vector field f_i as long as $\Sigma(i,x,\eta,u) = i$. When $\Sigma(i,x,\eta,u)$ becomes equal to $j \neq i$, the system dynamics switches to (M_j, f_j). The value of $\Sigma(i,x,\eta,u)$ can change either because the trajectory of the system leaves the manifold M_i and enters M_j, or because the discrete input η changes. In general, the vector fields in $\mathcal{F}$ will be different, reflecting changes in the dynamics of the system. Also the dimensions of the manifolds in $\mathcal{M}$ might be different. In this paper we assume that there are finitely many switches in any finite time interval. We therefore exclude phenomena like chattering.

3 Passivity

We start by reviewing the notion of passivity for smooth systems. We follow the development in [12]. A system defined by:

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x,u) \end{aligned} \tag{2}$$

where $f(0,0) = 0$ and $h(0,0) = 0$ is *passive* if there exists a $\mathcal{C}^1$ positive semidefinite function $V(x)$ (called the storage function) such that:

$$u^T y \geq \frac{dV}{dt} + \varepsilon u^T u + \delta y^T y + \rho \psi(x) \quad \forall (x,u) \tag{3}$$

where $\varepsilon, \delta, \rho$ are nonnegative constants, and $\psi(x)$ is a positive semidefinite function of x such that

$$\psi(x(t)) \equiv 0 \Rightarrow x(t) \equiv 0.$$

If $\varepsilon > 0$ the system is *input strictly passive*, if $\delta > 0$ the system is *output strictly passive*, and if $\rho > 0$ the system is *state strictly passive*.

The intuitive interpretation of this definition is that passive systems can not generate energy on their own. It can be shown that if the system is state strictly passive the origin is an asymptotically stable equilibrium point, and the storage function V becomes a Lyapunov function. But what makes passivity so useful for stability analysis is that, loosely speaking, an interconnection of passive systems is again passive. This observation has been for example the basis of the stability proofs for teleoperation systems [3, 4] and haptic devices [1, 9, 10, 14].

However, even if the concept of passivity and the energy considerations that lead to stability are intuitive and therefore appealing, the concept might be quite misleading when dealing with hybrid systems. It would seem reasonable to conclude that if the system can switch between two sets of state equations and if each set of equations defines a passive system, the resulting hybrid system must also be passive. It is easy to construct a counterexample that shows that such a conclusion would be wrong [18]. A sufficient condition for a hybrid system that switches between two passive systems to be stable is that the storage functions of both systems are identical. This requirement is overly restrictive for hybrid systems and we thus propose an alternative, less stringent approach.

It has been shown in [6, 11, 13, 16] that it is not necessary to find a global Lyapunov function in order to guarantee that a hybrid system is stable; it suffices to analyze the stability in each dynamic regime (M_j, f_j) and impose additional conditions that need to hold when the system switches. It is known that the storage function of a passive system is a candidate Lyapunov function for stability analysis. This suggests that the passivity for hybrid systems should be defined in terms of storage functions of the individual discrete regimes; requiring that a single global storage function exists would be too strict.

The stability tests in [6, 16] focus on the continuous dynamics of the system, no properties of the discrete dynamics are exploited. It is thus worthwhile to formulate a stability criterion in which the discrete dynamics plays a more prominent role. Such a criterion was proposed in [19] and we briefly summarize it here.

Let $\Omega = (\Xi, \mathcal{M}, \Gamma, \mathcal{U}, \Sigma, \mathcal{F}, \mathcal{H})$ be a hybrid system. The discrete dynamics Σ induces a relation $\sigma(\Xi)$, if we put $\sigma(i,j)$ when it is possible to switch from the manifold M_i to the manifold M_j. More formally:

$$\sigma(\Xi) = \{(i,j) \mid \exists x \in M_i,\ \eta \in \Gamma,\ u \in \mathcal{U} \text{ s.t. } \Sigma(i,x,\eta,u) = j\} \tag{4}$$

The relation σ represents the switching behavior of a hybrid system and is useful for studying its stability. More precisely, if a partial order can be extracted from σ, it can be interpreted in a certain sense as a discrete equivalent of the Lyapunov function.

Assume a controller has been designed for each of the dynamic regimes. Therefore, for each manifold $M_i \in \mathcal{M}$, there is a controller g_i:

$$g_i : M_i \to \mathcal{U} \tag{5}$$

We will assume that the function g_i is measurable, but it can be discontinuous. While the controllers g_i determine the continuous evolution of the system, the evolution of the discrete state also depends on the discrete input η. It is therefore also necessary to design a discrete controller:

$$\mathcal{S} : \Xi \times \mathbb{R}^n \to \Gamma \tag{6}$$

which at each state $(i,x) \in \Xi \times \mathbb{R}^n$ selects a discrete input.

Once the discrete controller and the continuous controllers have been designed, the hybrid system becomes autonomous. The function Σ thus only becomes a function of the discrete state i and the continuous state x. The following result, similar to that proved in [19], provides a sufficient condition for the stability of the resulting system:

Proposition 1 *Let Ω be a controlled hybrid system and let $\sigma^{\text{Trans}}(\Xi)$ be a transitive closure of the relation σ defined by Equation (4). If the following conditions hold:*

1. *For each regime i there exists a Lyapunov function V_i such that $V_i(e_i) = 0$ (e_i is thus an asymptotically stable equilibrium point for the regime i).*
2. *There exists a partial order $\preceq$ within $\sigma^{\text{Trans}}(\Xi)$ such that the set of minimal elements of $\preceq$ coincides with $\mathcal{E}$, where $\mathcal{E}$ is the set of those regimes for which $e_i = 0$.*
3. *For each $i \notin \mathcal{E}$, there exists $L_i > 0$ such that $\Sigma(i,x) \prec i$ for every $x \in B(e_i, L_i) \cap M_i$. For $i \in \mathcal{E}$, there exists $L_i > 0$ such that $\Sigma(i,x) \in \mathcal{E}$ for every $x \in B(0,L) \cap M_i$.*
4. *$V_i(x(t_{i,k})) + \Delta \le V_i(x(t_{i,k-1}))$, where $t_{i,k}$ denotes the k-th time that the vector field f_i becomes "active" (i.e., $\xi(t_{i,k}^-) \neq \xi(t_{i,k}^+) = i$) and $\Delta > 0$ is a fixed constant.*

Then the system Ω globally converges to the origin.

Remark 1 *The stability test in [6] is a special case of Proposition 1 when all the regimes of the system belong to $\mathcal{E}$ and $\Delta = 0$.*

In the case of smooth systems, the definition in Equation (3) is related to the Lyapunov method for stability analysis. Similarly, a test for stability analysis of hybrid systems can be used to extend the notion of passivity to hybrid systems. Proposition 1 suggests the following definition of passivity:

Definition 2 *Let Ω be a controlled hybrid system and let $\sigma^{\text{Trans}}(\Xi)$ be a transitive closure of the relation σ defined by Equation (4). The system will be called a* ***passive hybrid system with partial order (PHSPO)*** *if the following conditions hold:*

1. *In each discrete regime i the system is state strictly passive. In other words, for each $i \in \Xi$, there exists a positive semidefinite storage function V_i such that $V_i(x) = 0 \quad \Leftrightarrow \quad x = e_i$, and a positive constant ρ_i such that*

$$u^T y \geq \frac{dV_i}{dt} + \rho_i \psi_i(x) \quad \forall (x,u) \tag{7}$$

   where (x,u) is a trajectory of (M_i, f_i).

2. *There exists a partial order $\preceq$ within $\sigma^{\text{Trans}}(\Xi)$ such that the set of minimal elements of $\preceq$ coincides with $\mathcal{E}$, where $\mathcal{E}$ is the set of those regimes for which $e_i = 0$.*

3. *For each $i \notin \mathcal{E}$, there exists $L_i > 0$ such that $\Sigma(i,x) \prec i$ for every $x \in B(e_i, L_i) \cap M_i$. For $i \in \mathcal{E}$, there exists $L_i > 0$ such that $\Sigma(i,x) \in \mathcal{E}$ for every $x \in B(0,L) \cap M_i$.*

4. *The storage functions V_i have the property:*

$$V_i(x(t_{i,k})) + \Delta \leq V_i(x(t_{i,k-1})) + \int_{t_{i,k-1}}^{t_{i,k}} u^T y dt, \tag{8}$$

   where $t_{i,k}$ denotes the k-th time that the vector field f_i becomes active and $\Delta > 0$ is a fixed constant.

Note that Equation (7) has to hold anytime the system is in the regime i, whereas the integral in Equation (8) runs over the regimes that the system traverses before switching back to i. This definition allows us to state the following result:

Proposition 2 *Consider a PHSPO according to Definition 2. If the storage functions $V_i(x)$ are positive definite then the zero-input system ($u(t) = 0$) converges to the origin $x = 0$.*

Proof: Since the system is PHSPO, $\frac{dV_i}{dt} \leq u^T y$ according to Equation (7). Therefore, if $u = 0$ the storage functions $V_i(x)$ are Lyapunov functions. For $u = 0$ they also satisfy all the conditions of Proposition 1 which implies that the system converges to the origin. ■

It is thus possible to use stability tests for hybrid systems to define the notion of passivity so that passivity guarantees stability. However, an extended notion of passivity will only be useful if it is possible to show that an interconnection of passive systems such as shown in Figure 1 will be passive. The following proposition shows that this is indeed the case.

Proposition 3 *Let $S_1 = (\Xi_1, \mathcal{M}_1, \Gamma_1, \mathcal{U}_1, \Sigma_1, \mathcal{F}_1, \mathcal{H}_1)$ and $S_2 = (\Xi_2, \mathcal{M}_2, \Gamma_2, \mathcal{U}_2, \Sigma_2, \mathcal{F}_2, \mathcal{H}_2)$ be two PHSPO that are interconnected as in Figure 1. Suppose that the feedback system has a well-defined model $S = (\Xi_1 \times \Xi_2, \mathcal{M}_1 \times \mathcal{M}_2, \Gamma_1 \times \Gamma_2, \mathcal{U}_1 \times \mathcal{U}_2, \mathcal{F}, \Sigma)$ with the continuous state $x = \left[x_1^T x_2^T\right]^T$, input $u = \left[{u_1}^T {u_2}^T\right]^T$, and output $h = \left[h_1^T h_2^T\right]^T$. Then S is a PHSPO.*

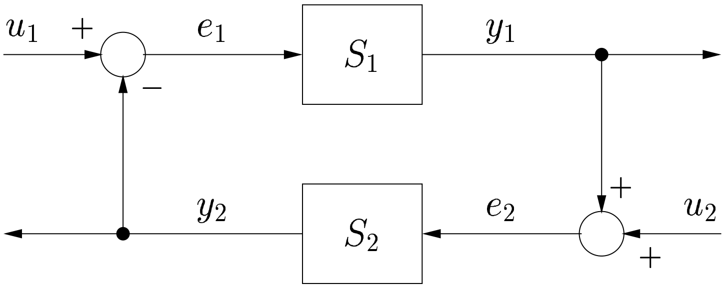

Figure 1: A feedback interconnection of control systems.

Proof: We will use superscripts or subscripts 1 and 2 to refer to S_1 and S_2, respectively. To show that S is a PHSPO we need to show that Equation (7) and all the conditions in Definition 2 hold. Note that the set of the discrete states of S equals $\Xi = \Xi_1 \times \Xi_2$. Let $(i_1, i_2) \in \Xi$. Since S_1 and S_2 are PHSPO, we have for $a = \{1, 2\}$:

$$e_a^T y_a \geq \frac{dV^a_{i_a}}{dt} + \rho^a_{i_a} \psi^a_{i_a}(x_a) \quad \forall (x_a, e_a)$$

From $e_1 = u_1 - y_2$ and $e_2 = u_2 + y_1$, we have:

$$e_1{}^T y_1 + e_2{}^T y_2 = u_1{}^T y_1 + u_2{}^T y_2 \tag{9}$$

Now define $V_{(i_1,i_2)}(x) = V^1_{i_1}(x_1) + V^2_{i_2}(x_2)$. From (9) and (9) we obtain:

$$u^T y \geq \frac{\partial V_{(i_1,i_2)}}{\partial x} \begin{bmatrix} f^1_{i_1}(x_1) \\ f^2_{i_2}(x_2) \end{bmatrix} + \min\{\rho^1_{i_1}, \rho^2_{i_2}\}(\psi^1_{i_1}(x_1) + \psi^2_{i_2}(x_2)) \tag{10}$$

which shows (7).

Next, observe that for S, $\Sigma = \Sigma_1 \times \Sigma_2$ and the relation σ equals $\sigma = \sigma_1 \times \sigma_2$. Recall that the later implies that $\sigma((i_1, i_2), (j_1, j_2))$ if and only if $\sigma_1(i_1, j_1)$ and $\sigma_2(i_2, j_2)$. It then follows that if we define $\preceq = \preceq_1 \times \preceq_2$, then $\preceq$ is a partial order within $\sigma^{\text{Trans}}(\Xi)$ whose minimal elements correspond to all the states (i_1, i_2) such that $e_{i_1} = e_{i_2} = 0$. It is easy to see that the next condition in 2 also holds.

Finally, we need to show that (8) holds for S. Consider $t_{(i_1,i_2),k}$ and $t_{(i_1,i_2),k-1}$, where $t_{(i_1,i_2),k}$ denotes the k-th time that the vector field $f_{(i_1,i_2)}$ becomes "active". Note that Equations (7) and (8) imply that if $t_1 < t_2$ and $\xi_a(t_1) = \xi_a(t_2) = i_a$, $V^a_{i_a}(x_a(t_1)) + \int_{t_1}^{t_2} u_a^T y_a dt \geq V^a_{i_a}(x_a(t_2))$ for $a = \{1, 2\}$. This is true regardless of the number and location of switches on the interval (t_1, t_2). But then it follows that

$$\begin{aligned}
V_{(i_1,i_2)}(x(t_{(i_1,i_2),k})) =& V^1_{i_1}(x_1(t_{(i_1,i_2),k})) + V^2_{i_2}(x_2(t_{(i_1,i_2),k})) \\
\leq& V^1_{i_1}(x_1(t_{(i_1,i_2),k-1})) + \int_{t_{(i_1,i_2),k-1}}^{t_{(i_1,i_2),k}} u_1{}^T y_1 dt + \\
& V^2_{i_2}(x_2(t_{(i_1,i_2),k-1})) + \int_{t_{(i_1,i_2),k-1}}^{t_{(i_1,i_2),k}} u_2{}^T y_2 dt - \min\{\Delta_1, \Delta_2\} \\
=& V_{(i_1,i_2)}(x(t_{(i_1,i_2),k-1})) + \int_{t_{(i_1,i_2),k-1}}^{t_{(i_1,i_2),k}} u^T y dt - \min\{\Delta_1, \Delta_2\}
\end{aligned}$$

■

The last two propositions lead to the following result:

Corollary 1 *If S_1 and S_2 are PHSPO and all the storage functions $V_i(x)$ are positive definite, then S is stable.*

4 Example

Consider a feedback connection of two systems S_1 and S_2 as shown in Figure 1. Let S_1 be a planar Cartesian haptic display (a 2 DOF gantry mechanism) and S_2 a model of a virtual environment. Let the model of S_1 be:

$$m\ddot{q}_1 + b\dot{q}_1 + kq_1 = u_1$$

where q_1 denotes the x and y coordinates of the haptic display, u_1 are the actuator forces, m is the mass , b is the friction in the linear bearings (the same in both directions) and k is the stiffness of the mechanism. By setting $x_1 = \left[q_1^T \ \ \dot{q}_1^T\right]^T$, we can write the following state equations:

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} x_1 + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u_1$$

$$y_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_1$$

The outputs of S_1 are the velocities in x and y directions. Since S_1 is a mechanical system with dissipation it is easy to see that it is passive where the storage function is the mechanical energy.

To illustrate the approach we let S_2 be a hybrid system with two states, $\Xi_2 = \{1, 2\}$. The discrete state ξ_2 evolves according to:

$$\xi_2(t^+) = \begin{cases} \xi_2(t^-) & V^{3-\xi_2(t_s^-)} + \int_{t_s}^t u_2{}^T y_2 dt < V^{3-\xi_2(t^-)} + \Delta \\ 3 - \xi_2(t^-) & \text{otherwise} \end{cases}$$

We have used the expression $3 - \xi_2$ to flip the discrete state between 1 and 2, and t_s to denote the time when the system last switched between the two regimes. Also, the function V_i is the storage function in the regime i. The switching rule therefore explicitly enforces the condition (8). Since each of the regimes is passive by itself so that (7) holds, the resulting system is a PHSPO. Such behavior might model for example a particle moving in a potential field that can switch between two configurations. If the human interacts with the system, the human input can be modeled as the input u_1 in Figure 1, with u_2 set to 0.

We would like to know whether the interaction between the haptic display and the environment will be stable. Since S_1 is a passive continuous mechanical system (and therefore trivially a PHSPO), and since S_2 was designed to be a PHSPO, the overall system is a PHSPO and therefore stable according to Corollary 1. Figure 2 shows a trajectory of the system. The first panel shows the evolution of the discrete state $\xi(t)$ and the input. The next panel shows the trajectory of the haptic device S_1. The third panel shows the trajectory of the system S_2. The initial states were $x_1 = \begin{bmatrix} 3 & 0 & 4 & 0 \end{bmatrix}^T$ and $x_2 = \begin{bmatrix} 5 & 3 \end{bmatrix}^T$.

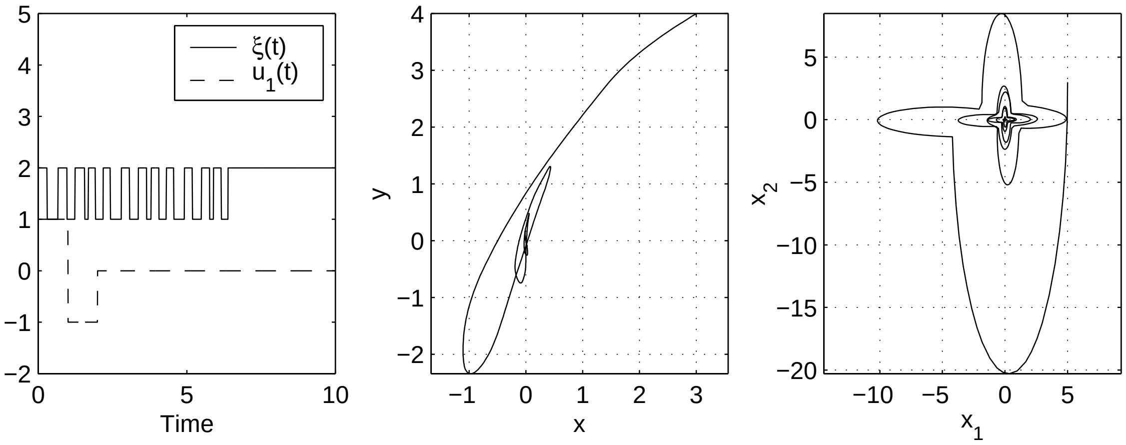

Figure 2: A haptic device interacting with a hybrid virtual environment.

5 Conclusion

We developed a framework for passivity analysis of hybrid systems. We showed that the classical notion of passivity is too restrictive in the hybrid systems setting and proposed a more general notion of passivity for hybrid systems. Several classical results linking passivity and stability were then generalized using stability criteria for hybrid systems. The work was motivated by problems in haptics and teleoperation where passivity is extensively used for stability analysis. An example demonstrates the applicability of the method.

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