Improving the Realism of Haptic Interaction for Teaching of Sensorimotor Skills

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Outline

Motivation: Dental Simulator

Visuo-Haptic Training Aids

Energy Based Haptic Rendering Algorithm

Conclusions
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Conclusions
Haptics and Sensorimotor Skill Acquisition

- Possible applications:
  - Teaching of sensorimotor skills
  - Collaborative haptics
  - Telemedicine
  - Rehabilitation
  - Guided haptic exploration of biological tissues
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- Haptic simulator can provide an alternative that:
  - is cost-effective;
  - allows 24/7 access;
  - provides convenient means to record and analyze performance.
Haptic Simulators in Medicine

**Haptic educational aids** are being developed in:

- diagnosis of prostate cancer (Burdea et al., 1999);
- surgical dissection training (Chial et al., 2002);
- laparoscopic surgery (Shen et al., 2005);
- endoscopy, anthroscopy, endovascular, etc.
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Tactile skills training in **dentistry**:

- time-consuming;
- requires one-on-one instructor-student interaction; instructor shortage of about 400 (ADEA);
- students are unable to feel what the instructor feels in the training process.
Other Dental Simulators vs. PerioSim

Existing dental simulators:

- Moog Dental Trainer (2008): Academic Centre for Dentistry Amsterdam (ACTA), Netherlands
- Voxel-Man DentalSurg (2009): University Medical Center Hamburg-Eppendorf, Germany

Primarily focused on drilling.
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Our simulator (**PerioSim**):

- designed for training first-year dental and hygiene students for periodontal probing and detection of caries;
- started as an interactive CD with no haptic capabilities;
- acts as a testbed system for investigating the problem of sensorimotor skill acquisition.
Simulator Components

- **Hardware:**
  - PHANToM Desktop 3-DOF haptic device;
  - PC with Intel Xeon 3 GHz CPU;
  - Crystal Eyes Workstation stereoscopic glasses.
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  - PC with Intel Xeon 3 GHz CPU;
  - Crystal Eyes Workstation stereoscopic glasses.

- **Software:**
  - custom developed software (C++);
  - GHOST API:
    - handles haptics;
    - provides the refresh rate of exactly 1000 Hz.
  - Coin3D API (OpenGL-based):
    - handles graphics;
    - provides the refresh rate of about 30 Hz on our hardware.
  - FLTK API for GUI elements;
  - anatomically accurate 3D polygonal model of a mouth and also of several instruments (VRML).
Simulator Setup
Simulator Structure

- **HARDWARE**
  - Haptic Device
    - Encoders
    - Actuators and Breaks
  - Desktop Computer

- **SOFTWARE**
  - Model Selection
    - Selection of new instruments, selection of objects for graphic and haptic control
  - Haptic Control Panel
    - Haptic properties, force indication
  - Graphic Control Panel
    - Transparency, navigation, viewpoints, object grouping, positioning aid tool
  - Record and Replay Functionality
    - Recording of procedures, observation/learning/testing modes, import and export of recordings
Simulator Structure

- over 10 dental instruments available for haptic use;
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- new instruments, gingiva, teeth and bone 3D models can easily be added;
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- new instruments, gingiva, teeth and bone 3D models can easily be added;
- the models are configured for use through a configuration file;
- instructional video aids are recorded to and played back from a platform-independent file format developed for storing visuo-haptic streams.
Simulator Functionality
Reality Validation

- User studies were conducted to assess the realism of the simulator (Steinberg et al., 2007)
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- 30 experienced dental and hygiene instructors were invited
- 4 aspects of the simulation were evaluated on a scale of 1 (lowest) to 7 (highest) and the following mean scores obtained:
  - Realism of the simulated oral image and instruments – 5.5...6.7 (for various objects)
  - Realistic feel of oral structures – 4.2...6.0 (for various structures)
  - Usefulness of the instruction layout – 6.3
  - Potential in teaching – 6.2
- Detailed breakdown of the results indicates that the simulated objects are quite realistic in both graphic and haptic aspects with the possible exception of gingival tissue
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Research Areas

- **Recording and playback of visuo-haptic streams** – several haptic augmentation schemes are possible ranging from full haptic playback to pseudo-playback schemes; evaluation of their effectiveness.
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- **Haptic rendering algorithms** – development of a framework for 6D haptic force rendering based on physics rather than heuristics.

- **PerioSim: Haptic virtual reality dental simulator** – testbed application for practical implementation and verification of theoretical results.
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Conclusions
Using Haptics in Skill Acquisition

- Two phases of teaching of a motor task:
  - **initial phase**: a teacher introduces a trainee to a new procedure;
  - **component strengthening phase**: a trainee practices the procedure he/she is already familiar with.
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- What is a sensorimotor skill? – two complementary components: position and force.

- Trajectory in both position and force spaces has to be followed with a certain degree of accuracy (haptic playback).
Using the Haptic Dimension

- Both components can be recorded

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- Any form of haptic playback should pursue two goals:
  - convey both the position and force data to the possible extent;
  - achieve educational objectives, i.e., it should be an aid in sensorimotor skill acquisition, not a distraction.
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- Solutions:
  - haptically display of one of the two trajectories while providing the information about the other through a different modality (Henmi and Yoshikawa, 1998);
  - present the position trajectory assuming that the correct force information will be obtained when the position error is small (Williams et al., 2004).
Haptic Playback

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- As long as the user follows the target with a reasonable degree of accuracy, he will feel the same **force** that was recorded and will reproduce the same **spatial trajectory**.
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- Assumption:
  User is cooperative and an active participant.
Haptic Playback

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- Haptic playback is approached as a **control problem**.
- As long as the user follows the target with a reasonable degree of accuracy, he will feel the same **force** that was recorded and will reproduce the same **spatial trajectory**.
- Assumption:
  - User is cooperative and an active participant.
- Trajectory of the visual target and the recorded trajectory may differ considerably.
Haptic Playback Implementation

Analogous to audio or video playback: the information is recorded and subsequently played back.
Haptic Playback Controller

Framework:

\[
\begin{bmatrix}
    p_0(t) \\
    F_{\text{act}}(t)
\end{bmatrix}
= \begin{bmatrix}
    p_d(t) \\
    -F_d(t)
\end{bmatrix} - W \begin{bmatrix}
    p(t) - p_d(t) \\
    F(t) - F_d(t)
\end{bmatrix},
\]

- Open loop controller:

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\begin{align*}
p_0(t) &= p_d(t) \\
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▶ Direct force controller:

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\begin{align*}
p_0(t) &= p_d(t) \\
F_{\text{act}}(t) &= - F_d(t) + K_F (F(t) - F_d(t))
\end{align*}
\]

\[K_F > 0\]
Control Strategies

- Coupled controller:

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p_0(t) = p_d(t)
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\(K_F > 0\)

- Crossed controller:

\[
\begin{align*}
    p_0(t) &= p_d(t) - K_P(F(t) - F_d(t)) \\
    F_{act}(t) &= -F_d(t) - K_F(p(t) - p_d(t))
\end{align*}
\]

\(K_P > 0, K_F > 0\)
Modified Coupled Controller

- In a coupled controller: \( p(t) = p_d(t) \Rightarrow F_{act}(t) = -F_d(t) \)
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Modified Coupled Controller

- In a coupled controller: \( p(t) = p_d(t) \Rightarrow F_{\text{act}}(t) = -F_d(t) \)
- Simplifying assumption: if the recorded force was the result of the interaction with the virtual environment, this same environment can be used to compute \( F_{\text{act}}(t) \)
- Position tracking is actively pursued by the guiding force \( F_{\text{gde}}(t) = K_F \Delta p(t) \) that drags the user towards the recorded position, and

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F_{\text{act}}(t) = -F_d(t) - F_{\text{gde}}(t)
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  F_{\text{act}}(t) = -F_d(t) - F_{\text{gde}}(t)
  \]
- Perfect position tracking results in an ideal haptic playback when both the position and the experienced force are identical to those experienced by the “expert”.
Error Computation

Position error defined as distance:

\[ \Delta p(t) = d(p_d(t), p(t)) \]
Error Computation

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\[ \Delta p(t) = d(p_d(t), p(t)) \]

- If the task is to follow a trajectory \( T = \{(t, p_d(t))\} \) in time:
  \[ d(p_d(t), p(t)) = \|p_d(t) - p(t)\|_2 \]
Error Computation

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- If the task is to follow a certain path $P = \{p_d\}$ in space and the task is not subject to time restrictions:
  $$d(p_d(t), p(t)) = \min_{q \in P} \{\|q - p(t)\|_2\}$$
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- Same works for force error
Guiding Force: Practical Implementation

To avoid “jumps” due to the guiding force changing the sign at the points where $p_d(t) \approx p(t)$, an additional condition is introduced:

$$\frac{\partial}{\partial d} F_{gde}(t, 0) = 0.$$
Guiding Force: Practical Implementation

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\]

One way to satisfy this condition:
Testing and Progress Monitoring

- Aggregate score $G$ reflects the user’s performance:

$$G = 100 - \frac{r_1}{N} \sum_{i=1}^{N} (\|\Delta p_i\|_2 + r_2 \|\Delta \alpha_i\|_2),$$

where $\Delta p_i$ and $\Delta \alpha_i$ are vectors of differences in position and orientation calculated on the $i$th frame.
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Guiding force is used to train the user to reduce the position error $\Delta p_i$.
Testing and Progress Monitoring

- Aggregate score $G$ reflects the user’s performance:

$$G = 100 - r_1 \frac{1}{N} \sum_{i=1}^{N} \left( \| \Delta p_i \|_2 + r_2 \| \Delta \alpha_i \|_2 \right),$$

where $\Delta p_i$ and $\Delta \alpha_i$ are vectors of differences in position and orientation calculated on the $i$th frame.

- Guiding force is used to train the user to reduce the position error $\Delta p$.

- User can be trained to reduce the orientation error $\Delta \alpha$ either by analogy through guiding torque or by a visual or audio alarm:

$$A(t) = \begin{cases} 
\text{ON}, & \| \Delta \alpha(t) \|_2 > \gamma, \\
\text{OFF}, & \| \Delta \alpha(t) \|_2 \leq \gamma,
\end{cases}$$

where $\gamma$ is an experimentally determined parameter.
Visuo-Haptic Streams: Summary

- New framework for **haptic playback** as a **control problem**
- **Linear** controller structure
- Direct force, coupled, crossed controllers
- Preliminary comparative evaluation of the controller performance indicate that the **crossed controller** is the most effective
- **User studies** are currently under way
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Conclusions
What is Haptic Rendering?

- **Haptic rendering** is the process of computing contact forces in a virtual environment which will be conveyed to the user of a haptic interface (Lin and Otaduy, 2008).
- Barriers for realistic haptic rendering:
  - **Hardware aspects** – availability of general purpose 6-DOF hardware devices.
  - **Algorithmic issues** – development of good haptic rendering algorithms both in terms of realism and computational complexity suitable for real-time implementation (ability to run at refresh rates of at least 500 Hz, Booth et al, 2003).
- Still an active research area
Overview of Existing Approaches

- **Penalty-based methods** – most notably, direct rendering; based on the penetration depth estimation (Moore and Wilhelms, 1995).
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- **Constraint-based methods** – based on constraining the motions of a substitute virtual object (Zilles and Salisbury, 1995); produce accurate results but are more difficult to implement in complex scenarios.
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- **Impulse-based techniques**
Direct Rendering

- **Object** $A$ is controlled by the user and **object** $B$ is another object in the virtual environment.

![Diagram showing Direct Rendering](image-url)
Direct Rendering

- **Object A** is controlled by the user and **object B** is another object in the virtual environment.

- When the collision is detected, penetration depth $e$ is estimated and **feedback force** $F$ is computed as a function of $e$, e.g. (Kim et al, 2003)

\[
F(t) = K_P e(t) + K_D \frac{de(t)}{dt}
\]

\[
T(t) = r \times F(t)
\]
Deficiency of Direct Rendering

Consider two different scenarios:

1) Initial position

2) Contact wrench $W = [F_T]$ that has only torque components.

- $e$
- $F$
- $T$
Deficiency of Direct Rendering

Consider two different scenarios:

1) Force perpendicular to the surface of the plane and a function of the translational penetration depth.

2) Contact wrench $W = [F T]$ that has only torque components.

Existing penalty-based methods $\Rightarrow$ force perpendicular to the surface of the plane and a function of the translational penetration depth.
Deficiency of Direct Rendering

- Consider two different scenarios:

- Existing penalty-based methods ⇒ force perpendicular to the surface of the plane and a function of the translational penetration depth.

- In reality, pure instantaneous torque ⇒ contact wrench \( W = [F \quad T]^T \) that has *only* torque components.
Proposed Energy-Optimal Algorithm

- Instead of computing translation-based penetration depth, full rigid-body configuration (translation and rotation) has to be taken into account.
Proposed Energy-Optimal Algorithm

- Instead of computing translation-based penetration depth, full rigid-body configuration (translation and rotation) has to be taken into account.

- Consider a system comprised of rigid bodies $A$ and $B$. The admissible configuration set $C$ is

$$C = \{ q \in Q | A(q) \cap B = \emptyset \},$$

where $Q = SE(3)$ is the configuration space and $A(q) = \{ \alpha \in \mathbb{R}^3 | \alpha \in A \text{ when } A \text{ is in configuration } q \in SE(3) \}$.
Configuration Space

\[ q_1, q_2, q_6 \in C \]
\[ q_3, q_4, q_5 \notin C \]
Rigid Body Motion

- Detailed description can be found in Murray et al, 1994.
- **Inertial** and **body** reference frames:
Formal Description of Rigid Body Motion

All possible configurations (including both position and orientation) of the rigid body form \( SE(3) \), the *special Euclidean* group in \( \mathbb{R}^3 \):

\[
SE(3) = \left\{ g \in \begin{bmatrix} R & d \\ 0_{1 \times 3} & 1 \end{bmatrix} \middle| R \in SO(3), d \in \mathbb{R}^3 \right\}.
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Formal Description of Rigid Body Motion

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- Each element $g \in SE(3)$ can be written as $g = e^{\Xi}$ for some twist $\Xi \in se(3)$, where

$$se(3) = \left\{ \Xi = \begin{bmatrix} \Omega & v \\ 0_{1 \times 3} & 0 \end{bmatrix} \mid \Omega \in so(3), v \in \mathbb{R}^3 \right\}.$$
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- Each $\Xi \in se(3)$ can be mapped onto a vector of twist coordinates $\xi = [v \quad \omega]^T \in \mathbb{R}^6$, where $\omega \in \mathbb{R}^3$ is given by $\Omega = \hat{\omega}$. Notation: $\Xi = \hat{\xi}$.
Exponential Mapping

\[ se(3) \sim \mathbb{R}^6 \]

\[ se(3) \rightarrow SE(3) \]

\[ \xi_g \rightarrow g \]

\[ \text{Exp} \]

\[ \text{Log} \]
Distance Function

- Suppose that bodies $A$ and $B$ are in collision and, thus, $A$ in some inadmissible configuration $p \in SE(3)$. 

$\text{d}(p, q)$ returns the distance between any inadmissible configuration $p$ and an admissible configuration $q \in C$. 

Distance function between configurations $p$ and $q$ is a kinematic metric function (Lin et al, 2000) on $SE(3)$. 

$Kane et al, 1999$
Distance Function

- Suppose that bodies $A$ and $B$ are in collision and, thus, $A$ in some inadmissible configuration $p \in SE(3)$.
- Assuming that $C \neq \emptyset$, define the **distance function** (Kane et al, 1999) as

$$d_C(p) = \min \{d(p, q) \mid q \in C\}.$$
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- $d(p, q)$ returns the distance between any inadmissible configuration $p$ and an admissible configuration $q \in C$.
- **Distance function** between configurations $p$ and $q$ is a kinematic metric function (Lin et al, 2000) on $SE(3)$. 

Distance Function

- For every inadmissible configuration $p$ a closest-point projection (Kane et al, 1999) $q$ can be found such that $q \in P_C(p)$, where

$$P_C(p) = \{ r \in C \mid d_C(p) = d(p, r) \} .$$

- What distance function to choose for the computation of $d(p, q)$?
Distance Function

- For every inadmissible configuration $p$ a closest-point projection (Kane et al, 1999) $q$ can be found such that $q \in P_C(p)$, where

$$P_C(p) = \{ r \in \overline{C} \mid d_C(p) = d(p, r) \}.$$

- What distance function to choose for the computation of $d(p, q)$? This function should:
  - be invariant to the choice of reference frames;
Distance Function

For every inadmissible configuration $p$ a **closest-point projection** (Kane et al, 1999) $q$ can be found such that $q \in P_C(p)$, where

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- What distance function to choose for the computation of $d(p, q)$? This function should:
  - be invariant to the choice of reference frames;
  - allow to obtain scale-invariant solutions;
  - be based on physics rather than heuristics and be theoretically justified.
Kinetic Energy as Distance Function

- Define the distance function as the kinetic energy \( T(p, q) \) needed to move the object between configurations \( p \) and \( q \) in a unit of time, i.e.

\[
d(p, q) = 2T(p, q) = \tilde{V}^T M \tilde{V},
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\( \tilde{V} \) – approximate body velocity in twist coordinates,
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- Instantaneous body velocity $V = g^{-1}(t)\dot{g}(t)$.

- Assuming that the trajectory of the rigid motion between configurations $p$ and $q$ is roughly linear, it can be shown that

$$\tilde{V} = \left[ \exp \left( \left[ -\frac{\xi p^{-1}q}{2} \right]^\wedge \right) p^{-1}(q - p) \right]^\vee.$$
Properties of Energy-Based Distance Function

Such distance function is

- bi-invariant, i.e.

\[ d(p, q) = d(ap, aq) = d(pb, qb), \]

where \( a, b \in SE(3). \)

\( \Rightarrow \) invariance to the choice of reference frames.

Bi-invariance \( \Rightarrow \) **well-defined** distance function (Lin et al, 2000).
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The motion between \( p \) and \( q \) requires the least amount of kinetic energy \( \Rightarrow \) this particular closest-point projection is **energy optimal**.
Wrench Computation

- Compute \( q = \arg \min_{r \in C} d(p, r) \).
Wrench Computation

- Compute $q = \arg\min_{r \in C} d(p, r)$.
- Error vector in twist coordinates ("6-D penetration depth")
  
  $e = \xi p^{-1} q$. 
Wrench Computation

- Compute $q = \arg \min_{r \in C} d(p, r)$.
- Error vector in twist coordinates ("6-D penetration depth")
  $e = \xi_{p^{-1}} q$.
- Feedback wrench is computed as a function of $e$ and translated into global coordinates.
Implementation Details

- Goal: to compute the **optimal** configuration
  
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- **Numerical** optimization-based techniques:
  - e.g. use gradient search to find \( q \in SE(3) \) or \( \xi_q \in \mathbb{R}^6 \);
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- **Analytical** approximation method:
  - exact analytical solution for a 2D case;
  - extension to a 3D case.
Analytical Solution – 2D Case

**Inadmissible** configuration \( p \) of a planar body after collision:
Analytical Solution – 2D Case

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Assumptions:

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- only one vertex is in collision.
Analytical Solution – 2D Case

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Assumptions:

- body is a convex polygon;
- only one vertex is in collision.

Optimal admissible configuration \( q \in PC(p) \):
2D Case – Distance Function

- Distance function,

\[ d = m(\beta - h)^2 + 4H_{3,3} \sin^2 \left( \frac{\theta - \varphi}{2} \right), \]

\( H_{3,3} \) is the (3, 3) element of the inertia tensor matrix \( H \) (used to compute matrix \( M \)).
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- Given parameters \( h \) and \( \varphi \), quantities \( \beta \) and \( \theta \) can be found such that the distance function is minimized.
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- Given parameters \( h \) and \( \varphi \), quantities \( \beta \) and \( \theta \) can be found such that the distance function is minimized.

- \((\beta, \theta)\) is a parametrization of configuration \( q \).
2D Case – Constraints

To find $\beta$ and $\theta$ the following constraints need to be satisfied:

$$\beta = -v_x \sin \theta - v_y \cos \theta,$$

$v_x$ and $v_y$ are $x$- and $y$-coordinates of vertex $v$ expressed in the body reference frame.
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\[ 0 \leq \theta \leq \pi - \gamma, \]

to prevent situations like this one:
2D Case – Optimization Functional

- Optimization of a functional

\[ J = m(-v_x \sin \theta - v_y \cos \theta - h)^2 + 4H_{3,3} \sin^2\left(\frac{\theta - \varphi}{2}\right). \]

\[
\frac{\partial J}{\partial \theta} \bigg|_{\theta = \tilde{\theta}} = m(v_x^2 - v_y^2) \sin 2\tilde{\theta} + 2mv_x v_y \cos 2\tilde{\theta} \\
+ 2mh(v_x \cos \tilde{\theta} - v_y \sin \tilde{\theta}) + 2H_{3,3} \sin(\tilde{\theta} - \varphi) = 0
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\]

- It can be shown that \( \tilde{\theta} = 2 \arctan x \), where \( x \) is a solution of equation

\[
A_1 x^4 + B_1 x^3 + C_1 x^2 + D_1 x + E_1 = 0,
\]
2D Case – Optimal Solution

where

\[ A_1 = mv_x v_y - mv_x h + H_{3,3} \sin \varphi, \]
\[ B_1 = 2(H_{3,3} \cos \varphi - mh v_y - mv_x^2 + mv_y^2), \]
\[ C_1 = -6mv_x v_y, \]
\[ D_1 = 2(H_{3,3} \cos \varphi - mh v_y + mv_x^2 - mv_y^2), \]
\[ E_1 = mv_x v_y + mv_x h - H_{3,3} \sin \varphi. \]
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Fast methods exist for finding closed form solutions of quartic equations. Optimal value \( \theta_0 \) in the general case is

\[ \theta_0 = \arg \min_{\theta} \{ T_{\theta=0}, T_{\theta=\tilde{\theta}}, T_{\theta=\pi-\gamma} \}. \]

and optimal values \( \beta_0 \) and \( q_0 \) could be determined.
Analytical Approximation – 3D Case

Consider the family of planes $\Pi = \{ P_\delta \mid \delta \in [0, \pi) \}$, each $P_\delta$ parametrized by its rotation angle $\delta = \angle(P_\delta, XY)$. 

![Diagram of planes and angles](image-url)
Analytical Approximation – 3D Case

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Each planar cross-section $P_\delta \cap A$ is a 2D body; a solution for a 2D case can be found.
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- Each planar cross-section $P_\delta \cap A$ is a 2D body; a solution for a 2D case can be found.

- **Optimal motion** of a 3D body corresponds to a motion along some $P_\delta$. 
3D Case – Combining 2D Solutions

- Internal angle $\gamma$ for each $\delta$ can be found from geometric considerations.
3D Case – Combining 2D Solutions

- Internal angle $\gamma$ for each $\delta$ can be found from geometric considerations.
- Let $G_{2D}(\delta) = \{\beta_\delta, \theta_\delta\}$ be the obtained 2D solution for a particular $\delta$. 
3D Case – Combining 2D Solutions

► Internal angle $\gamma$ for each $\delta$ can be found from geometric considerations.

► Let $G_{2D}(\delta) = \{\beta_\delta, \theta_\delta\}$ be the obtained 2D solution for a particular $\delta$.

► Augmenting the set $G_{2D}(\delta)$ by the axis of rotation $\omega_\delta = [\sin \delta \ 0 \ \cos \delta]^T$ the same motion can be expressed in 3D as $G_{3D}(\delta, G_{2D}(\delta)) = \{\beta_\delta, \theta_\delta, \omega_\delta\}$. 
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- For each $G_{3D}(\delta, G_{2D}(\delta))$ there is a corresponding value of optimal 3D configuration $q(G_{3D}(\delta, G_{2D}(\delta)))$ and the distance metric $d(p, q(G_{3D}(\delta, G_{2D}(\delta))))$. 

$\gamma$ for each $\delta$ can be found from geometric considerations.
3D Case – Optimal Solution

- The optimal value $\delta_0$ with respect to the distance metric is determined by

$$\delta_0 = \arg \min_{\delta \in [0, \pi]} d(p, q(G_{3D}(\delta, G_{2D}(\delta)))).$$
3D Case – Optimal Solution

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$$
\delta_0 = \arg \min_{\delta \in [0, \pi)} d(p, q(G_{3D}(\delta, G_{2D}(\delta))))
$$

- In practice it is required to discretize the domain of $\delta$ and thus work with an approximation such as

$$
\delta_0 \approx \arg \min_{\delta \in \Sigma} d(p, q(G_{3D}(\delta, G_{2D}(\delta))))
$$

where $\Sigma = \left\{ \frac{n\pi}{N_{\delta}} \middle| n = 0, 1, \ldots, N_{\delta} - 1 \right\}$ and $N_{\delta}$ is the number of discretization points.
Analytical Approximation – Performance Analysis

Proposed method consists of three stages:

- Computation of a 2D solution. Query rate $f_1 = 60$ kHz for 1 GHz CPU and roughly constant.
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- Procedure for combining the family of 2D solutions into a 3D optimal solution. Query rate is very high compared to the other two stages, so this stage can generally be neglected.
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Combined query rate $f = \left( f_1^{-1} + f_2^{-1} \right)^{-1} = 47$ kHz. If running the algorithm $N_\delta$ times and $f_0 \geq 1$ kHz, then $f_0 \leq \frac{f}{N_\delta}$, therefore $N_\delta \leq 47$ and $\Delta\delta \geq 3.8^\circ$. 
Numerical Example – Static

\[
\begin{bmatrix}
-2.5000 \\
24.9166 \\
0 \\
0 \\
-0.2000
\end{bmatrix}
, \quad
\begin{bmatrix}
-1.1737 \\
27.0543 \\
0 \\
0 \\
-0.0867
\end{bmatrix}
, \quad
\begin{bmatrix}
-0.2959 \\
2.0510 \\
0 \\
0 \\
0.1133
\end{bmatrix}
, \quad W = f(e).
\]
Numerical Example – Dynamic

Location of the center of mass:
Numerical Example – Dynamic (cont.)

$Y$-projection of force:
Numerical Example – Dynamic (cont.)

Torque in $XY$-plane:
Energy Based Haptic Rendering: Summary

- Proposed method provides a theoretical basis for realistic haptic rendering of 6-DOF virtual environments.
- Based on the analysis of full configuration of a rigid body.
- Produces forces and torques that are more appropriate for haptic simulation.
- Implementation: direct numerical optimization or analytical approximation.
- Closed form solution for a planar case and the solution for the general case through a one-dimensional search.
Outline

Motivation: Dental Simulator

Visuo-Haptic Training Aids

Energy Based Haptic Rendering Algorithm

Conclusions
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- **PerioSim** – haptic dental simulator currently used by first-year dentistry students at the University of Illinois at Chicago
- URL: [http://www.dentaltrainingsimulator.com](http://www.dentaltrainingsimulator.com)
- A possible implementation of a way to convey haptic data from the instructor to a student is suggested in the form of visuo-haptic training aids.
- Several **haptic playback** controllers may be used depending on the application.
- New energy based **haptic rendering** framework provides a theoretical basis for realistic haptic rendering of 6-DOF virtual environments.
Publications


Publications


See these and more at CVRL website: [http://www.cvrl.cs.uic.edu/](http://www.cvrl.cs.uic.edu/)
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