Location control for information dissemination

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Abstract—The paper investigates location control for information dissemination, building on our previous work [1] on coverage with information aggregation. In this scenario, information generated in the area needs to be communicated to multiple destinations. The area is partitioned into subregions within which only one node provides service. The main contribution of this work is an algorithm for optimal placement of the nodes. We define the information dissemination density function over the area that combines information generating and information receiving densities. Subsequently, we show that the cost of information dissemination only explicitly depends on information generating and receiving density. Finding the optimal node location for information dissemination is formalized by an optimization problem with respect to the partition of the area and location of all nodes. We analyze the optimality conditions for both the partition and location and explicitly derive the optimal location for all nodes given any partition of the area. We also design a distributed iterative motion control algorithm in discrete time that drives all nodes to an optimal service configuration from any arbitrary initial configuration. We prove that under our control algorithm, the cost function decreases along iterations and converges to a local minimum. Numerical simulations that demonstrate the effectiveness of our motion control algorithm are also provided.

I. INTRODUCTION

Coverage control is one of the most important tasks in mobile robot networks [2]–[5]. In a mobile robot network, each mobile robot, also known as mobile node, is responsible for providing certain service in its assigned subregion. Usually, a locational optimization problem is solved to determine both the optimal assignment of subregions and optimal locations of all mobile robots [3], [6]–[10]. Under this problem formulation, a Lloyd-like gradient-descent control algorithm is an effective method for the network to achieve optimal service configuration [1], [2].

One of the most common services a mobile robot network provides is data collection [11], [12]. In [1], [13], [14], the authors investigate the problem when the network performs both coverage and communication service. The minimization problem is formalized by adding an appropriate term to the conventional coverage cost function. In these works, all messages generated in the network are sent to a single sink node.

In our work, we consider a group of mobile nodes performing information dissemination over an area. Information generated at any location is distributed to multiple locations in the same area governed by a distribution function. For example, in a battlefield, information obtained by different units needs to be aggregated to the command center, processed by the center and distributed to appropriate combat units. The area is partitioned into subregions so that only one node provides service within each subregion. The rate of information generated over the area is governed by the information generating density function. The network has a star structure where mobile nodes connect to a sink node directly, but mobile nodes do not have direct connection to each other. Any information that needs to be distributed is first collected and sent to the sink node by a mobile node that serves the subregion. Then the sink node processes the information and forwards it to an appropriate mobile node whose dominant subregion covers the destination. In the end, the mobile node sends the information to the destination. We try to minimize the total cost of information dissemination with respect to both the partition of the area and locations of all nodes.

Motivated by the cost of information aggregation in [1], we model the cost of sending information as a product of the amount of information and the distance between two locations. Hence, the cost of information dissemination is an integral of the above cost over all information generated in the area. Coverage with information aggregation studied in [1] becomes a special case of information dissemination where coverage cost is interpreted as a bottom level information dissemination from all points over the area to their appropriate local nodes. When information generates at a point, we define the information dissemination function that describes how much information is sent where. Based on the dissemination function, we are able to compute the information receiving density function over the area. Parallel to the analysis in [1], we analytically determine the equilibrium location for all nodes. We also determine the optimal partition of the area given the location of all nodes. Using these two results, we propose a distributed iterative gradient-descent motion control algorithm in discrete time that is able to drive all nodes to an equilibrium location from any arbitrary initial location.

II. PRELIMINARIES

In this section, we first review two types of space partition following [7]: the Voronoi diagram and the power diagram. We then review the locational optimization problem of coverage control with information aggregation following [1]. We show that coverage, formally, can be viewed as one type of information aggregation. In the end, we generalize information aggregation to information dissemination, where the destination of information is no longer a single sink node, it can be anywhere in the interested area.

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A. The Voronoi and power diagrams

Partitioning a non-empty 2D space $\Omega$ into $n$ subregions with $n$ generators located at $P = [p_1, p_2, \ldots, p_n]$ is equivalent to determining an assignment rule $\delta(q, p_i)$, $\forall q \in \Omega$ as a mapping from $\Omega \times P$ to $\{0, 1\}$, such that

$$
\delta(q, p_i) = \begin{cases} 
1 & \text{if } f(q - p_i) \leq f(q - p_j), \\
0 & \text{if } j \in [1, 2, \ldots, n], j \neq i,
\end{cases}
$$

where $f(\cdot)$ is a distance function defined over $\Omega$. Clearly, we need the function $f(\cdot)$ to be both non-negative and non-decreasing. Under the assignment rule (1), the collection of points assigned to the generator $p_i$ is

$$W(p_i) = \{q \mid \delta(q, p_i) = 1, q \in \Omega\}.$$ 

For the Voronoi diagram, function $f(\cdot)$ is the Euclidean distance function $f(\cdot) = ||\cdot||$. The power diagram is a natural generalization of the Voronoi diagram. The distance becomes the additive weighted power distance function $f(\cdot) = ||\cdot||^2 + w$, where $w$ is a constant.

Remark 1: Unlike Voronoi cells, a power cell may become empty due to unbiased assignment to $w_i$. When a power cell is not empty, it must be a convex polygon. Sometimes, due to the same reason, a generator of a power cell might not be located in the cell.

B. Coverage control with information aggregation

In [1], the authors study a problem where in addition to covering a convex area $\Omega$, $n$ mobile sensor nodes are designed to send measured information of an event to a sink node that aggregates information. The problem is to determine a partition $W = [W_0, W_1, W_2, \ldots, W_n]$ of the area and locations of all sensor nodes including the sink $P = [p_0, p_1, p_2, \ldots, p_n]$ that minimize the cost function:

$$J = J_{cov} + \alpha J_{agg},$$

where the coverage cost is

$$J_{cov}(P, W) = \sum_{i=0}^{n} \int_{W_i} ||q - p_i||^2 \phi(q) dq$$

and the information aggregation cost is

$$J_{agg}(P, W) = \sum_{i=0}^{n} \left( ||p_i - p_0||^2 \cdot \int_{W_i} \phi(q) dq \right).$$

In the cost function, $\phi(q)$ denotes how often the event happens over the area. In the above problem, it is assumed that the sink node performs both information aggregation and coverage.

The authors also derive the optimality conditions for both the partition of the area and the locations of all sensor nodes. They design a Lloyd type iterative motion control algorithm that drives all sensor nodes to the optimal sensing configuration from any arbitrary initial configuration. Within each iteration, a power diagram is first created that minimizes the cost function given the location of all sensor nodes from the last iteration. Then, all sensor nodes move to new locations that minimize the cost function given the newly updated power diagram. The authors provide rigorous proof that by following the iteration, the cost function converges to a local minimum. The global optimal solution yet remains an open question.

Remark 2: However, the coverage cost (3) can actually be interpreted as information aggregation cost for aggregating information generated by any end point to its local aggregator. The original information aggregation cost (4) is one level higher than the local information aggregation cost. It aggregates information from local aggregators to the sink node. The constant $\alpha$ in the total cost (2) reflects the difference of aggregation efficiency in two levels. Generally, in the higher level, it is more energy efficient to aggregate the same amount of information over the same distance. Here, we see a hierarchical information aggregation process implemented over the area: from end locations to local nodes then to the sink node.

C. Information dissemination

The optimal sensor placement problem for coverage control with information aggregation can now be viewed as optimal node placement problem in hierarchical information aggregation. In [1], the authors assume the only destination of information is the sink node. In other applications, such as multi-point wireless communication, a user at a certain location generates multiple messages that need to be sent to users at different locations. In such a case, information is sent to locations other than just the sink node. We call this process information dissemination. Obviously, information aggregation where the destination of information is restricted to the sink node, is a special case of information dissemination.

We assume that information is generated over a convex area $\Omega$ with a density function $\phi(q)$ that describes how frequently information is generated at each point. We assume that how much information is sent to where depends on an information dissemination function $s(q, q_0)$:

$$\int_{\Omega} s(q, q_0) dq = 1,$$

$$s(q, q_0) \geq 0, q_0 \in \Omega.$$ 

Information dissemination is performed by a mobile sensor network that has a star structure under which all mobile sensor nodes connect to the sink node directly. The $n + 1$ sensor nodes are located at $P = [p_0, p_1, \ldots, p_n]$, where $p_0$ corresponds to the sink node. The area is partitioned into $n + 1$ subregions, $W = [W_0, W_1, \ldots, W_n]$, so that only one sensor node provides service within each subregion. Information generated at location $q \in W_i$ is first aggregated to node $p_i$. Then the information is aggregated to the sink node $p_0$ regardless of whether the information destination is within the same subregion or not. This is because that the information needs to be appropriately processed by the sink node before it can be used. It also simplifies the design
of routing algorithms implemented on mobile sensor nodes. When the sink node receives the information, it processed the information and forwards it to appropriate mobile sensor nodes that cover the final destinations. Finally, that local node forwards the information to the destination, see fig. 1.

Parallel to the cost of information aggregation, we define the cost of information dissemination as

$$J_{dis}(P, W) = \sum_{i=0}^{n} \int_{W_i} ||q - p_i||^2 \phi(q) dq + \alpha \sum_{i=0}^{n} \left( ||p_i - p_0||^2 \cdot \int_{W_i} \int_{W_i} \phi(q) dq \right) + \alpha \sum_{i=0}^{n} \left[ ||p_i - p_0||^2 \cdot \int_{W_i} \int_{W_i} \int_{W_i} \phi(q) s(q,q_i) dq dq_i dq_i \right] + \sum_{i=0}^{n} \int_{W_i} ||q_i - p_i||^2 \left( \int_{W} \int_{W} \phi(q) s(q,q_i) dq dq_i \right) dq_i. \quad (5)$$

The first two terms in the above cost function penalize aggregating information from end points to the sink node. The last two terms penalize sending information from the sink node to all destinations.

Remark 3: Note that the first two items in the cost function are almost the same as the last two except for the density function. Actually they penalize the two inverse processes. The cost of both information sending and receiving is independent on the direction of information flows. It only depends on the distance and amount of information being sent.

Remark 4: The cost of transmitting the same amount of information over the same distance at different levels in the network may be different. Hence, there appears a constant $\alpha$ in the cost function that weighs the cost of information sent from local nodes to the sink node. Usually, a higher level transmits information in a more energy efficient way than lower levels do. Hence, the constant $\alpha < 1$.

III. Equilibrium Analysis of Information Dissemination

In this section, we first formalize the motion control problem for information dissemination as a minimization problem with respect to two sets of variables: the partition of the area and the location of all mobile nodes. We then derive the optimality conditions for both sets of variables with the other set fixed.

A. Problem formulation

In the last section, we built a framework describing the process of information dissemination over a convex area $\Omega$ and determined its cost in terms of the information generating density $\phi(q)$, information dissemination function $s(q, q_i)$, locations of mobile sensors $P = [p_0, \cdots, p_n]$ and partition of the area $W = [W_0, \cdots, W_n]$. We point out that coverage with information aggregation in [1] fits within our information dissemination framework: it is a special case of information dissemination where the destination of information is the sink node.

In an information dissemination scenario, all $n$ local nodes connect to the sink node directly, which means the network has a star structure. The area is partitioned into subregions in which only one sensor node provides service. Information generated at a certain location $q$ is first sent to a local node whose dominant subregion $W_i$ covers $q$. Because local nodes do not have direct connections between each other (the information needs to be processed before it can be used), when the local node receives the information from end points, it forwards it to the sink node where the information is processed and it is routed to an appropriate local node whose dominant subregion covers the final destination. We study how to find the configuration that minimizes the dissemination cost with respect to both the partition of the area and locations of sensor nodes that can minimize the cost function.

Problem statement: Assume $n$ mobile nodes and one mobile sink node are providing information dissemination service over a convex area $\Omega$. The rate of information generation at point $q$ is governed by a semi-positive density function $\phi(q)$. The dissemination of all information generated at $q$ is described by a sending distribution function $s(q, q_i)$. Let $P = [p_0, p_1, \cdots, p_n]$ be the location of each node and $W = [W_0, W_1, \cdots, W_n]$ be the partition of the area $\Omega$, where $p_0$ and $W_0$ corresponds to the sink node. The optimal node placement problem for information dissemination is to determine the solution of the following minimization problem:

$$\min_{P, W} J_{dis}(P, W),$$

where $J_{dis}$ is defined in (5).

B. Information receiving density function

Based on the sending distribution function $s(q, q_i)$ and information generating density $\phi(q)$, we are able to define
the information receiving density, \( r(q) \), for every point in \( \Omega \):

\[
 r(q) = \iiint_{\Omega} \phi(q)s(q, q_t) \, dq_t.
\]

This equation reveals the relation between information generating and receiving. Since all information generated in \( \Omega \) is sent to destinations also in \( \Omega \), we conclude that the following equation holds true:

\[
 \iiint_{\Omega} \phi(q) \, dq = \iiint_{\Omega} r(q) \, dq.
\]

Now, the cost function (5) can be written as:

\[
 J_{\text{dis}}(P, W) = \sum_{i=0}^{n} \int_{W_i} \left( \|q - p_i\|^2 + \alpha \|p_i - p_0\|^2 \right) \cdot \left( \phi(q) + r(q) \right) \, dq
\]

(7)

**C. Optimizing information dissemination**

Note that by defining information receiving density function, the cost function of information dissemination can be written as (7). The optimization problem reduces to the coverage control with information aggregation problem in [1]. Instead of having a density function \( \phi(q) \), the density function in this section is a combination of both the rate of information generating and receiving. By setting the receiving density \( r(q) = 0 \), our cost function penalizes only hierarchical information aggregation. Since \( \phi(q) \) is semi-positive and \( r(q) \) is semi-positive over \( \Omega \), \( \phi(q) + r(q) \) is also semi-positive over \( \Omega \). Hence, the optimality conditions for both the partition of the area and location of each node are parallel to those in [1].

First, we find out the optimal partition of the area given the locations of all nodes. The problem becomes the following minimization problem:

\[
 \min_{W} J_{\text{dis}}(P^0, W)
\]

given the location of each node at \( P^0 \). The integrand of the cost function in (7) is

\[
 \left( \|q - p_i\|^2 + \alpha \|p_i - p_0\|^2 \right) \cdot (\phi(q) + r(q)).
\]

Using the same method in [1], we conclude that the optimal partition of the area is the power diagram generated by the location of each node.

Second, we find out the optimal location of each node given the partition of the area. In this case, the problem becomes the following minimization problem:

\[
 \min_{P} J_{\text{dis}}(P, W^0)
\]

given the partition of the area \( W^0 \). The first order necessary condition and second order sufficient condition for \( P \) minimizing \( J_{\text{dis}} \) given \( W^0 \) are \( \nabla J = 0 \) and \( \nabla^2 J > 0 \), respectively.

Before we compute the gradient, we need to define the mass \( M_i \) and centroid \( C_i \) of each subregion in terms of information generating and receiving density and the partition of the area. Let \( M_i = \frac{1}{n_i} \int_{W_i} (\phi(q) + r(q)) \, dq \) and \( C_i = \frac{1}{\int_{W_i} (\phi(q) + r(q)) \, dq} \int_{W_i} (\phi(q) + r(q)) \, dq \). Parallel to the results in [1], the gradient of \( J_{\text{dis}} \) with respect to \( P \) is:

\[
 \nabla J_{\text{dis}} = \left[ \frac{\partial J_{\text{dis}}(P, W)}{\partial p_0}, \ldots, \frac{\partial J_{\text{dis}}(P, W)}{\partial p_n} \right],
\]

where

\[
 \frac{\partial J_{\text{dis}}(P, W)}{\partial p_0} = \sum_{i=0}^{n} \int_{W_i} \frac{\partial \left( \|q - p_i\|^2 + \alpha \|p_i - p_0\|^2 \right)}{\partial p_0} (\phi(q) + r(q)) \, dq
\]

\[
 = 2p_0M_0 - 2C_1M_1 + \sum_{i=1}^{n} 2\alpha(p_0 - p_i)M_i
\]

\[
 = 2p_0 \left( M_0 + \alpha \sum_{i=1}^{n} M_i \right) - 2 \left( C_0M_0 + \alpha \sum_{i=1}^{n} p_iM_i \right)
\]

(9)

and

\[
 \frac{\partial J_{\text{dis}}(P, W)}{\partial p_i} = \sum_{i=0}^{n} \int_{W_i} \frac{\partial \left( \|q - p_i\|^2 + \alpha \|p_i - p_0\|^2 \right)}{\partial p_i} (\phi(q) + r(q)) \, dq
\]

\[
 = (2 + 2\alpha)p_iM_i - 2(C_i + \alpha p_0)M_i.
\]

(10)

The Hessian matrix \( \nabla^2 J \) is

\[
 2^j \begin{pmatrix} M_0 + \alpha \sum_{i=1}^{j} M_i & -\alpha M_1 & \cdots & -\alpha M_n \\ -\alpha M_1 & (1 + \alpha)M_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\alpha M_n & 0 & \cdots & (1 + \alpha)M_n \end{pmatrix}
\]

Because the mass of each subregion is positive, we conclude that the \( j^{th} \) leading principal minors of the Hessian matrix,

\[
 2^j \left( M_0 + \alpha \sum_{i=1}^{j} M_i \right) \cdot \prod_{i=1}^{j} \left( \frac{M_0 + \alpha \sum_{i=1}^{j} M_i}{M_0 + \alpha \sum_{i=1}^{j} M_i} \right)^2
\]

are all positive as well.

Therefore, we draw the conclusion that the solution of \( \nabla J = 0 \) minimizes the cost function \( J_{\text{dis}}(P, W^0) \) given the partition of the area \( W^0 \). We solve (9) and (10) for \( p_0 \) and \( p_i \) respectively,

\[
 p_0^* = \frac{C_0M_0 + \alpha \sum_{i=1}^{n} p_iM_i}{M_0 + \alpha \sum_{i=1}^{n} M_i},
\]

(11)
and
\[ p^*_i = \frac{C_i + \alpha p^0_i}{1 + \alpha}. \] (12)

Though the above solutions are coupled in \( p^0_0 \) and \( p^*_i \), it is easy to decouple them. The decoupled solutions are
\[ p^*_0 = \frac{C_0 M_0 + \alpha \sum_{i=0}^{n} C_i M_i}{M_0 + \alpha \sum_{i=0}^{n} M_i} \] (13)
and
\[ M_0(C_i + \alpha C_0) + \alpha C_0 \sum_{i=0}^{n} M_i + \alpha^2 \sum_{i=0}^{n} C_i M_i \]
\[ p^*_i = \frac{1 + \alpha}{1 + \alpha}. \] (14)

Remark 5: Both coupled and decoupled solutions have their own advantages. If a central computed algorithm is implemented, both solutions work fine. If a distributed computed algorithm is implemented, it is better to obtain \( p^0_0 \) from decoupled solution (13) and \( p^*_i \) from coupled solution (12), respectively. In distributed implementation, every node is responsible for the computation of its own mass \( M_i \) and centroid \( C_i \). \( p^*_i \) can be obtained by requiring mass and centroid from all local nodes. Then \( p^*_i \) can be obtained by requiring \( p^0_0 \) from the sink node. If we use a decoupled solution to compute \( p^*_i \), every local node will require both mass and centroid from all other nodes in the network, which results in an all-to-all communication.

IV. DISTRIBUTED MOTION CONTROL FOR OPTIMIZING INFORMATION DISSEMINATION

In this section, we develop a motion control algorithm that is able to drive all mobile nodes (including the sink node) to the optimal locations that minimize the information dissemination cost function (7). Our algorithm is a distributed Lloyd-type iterative algorithm implemented in discrete time. We prove that by applying our control algorithm, the cost function decreases monotonically along the iterations and eventually converges to a local minimum.

Based on the equilibrium analysis, we are able to find a partition of the area \( W \) that minimizes the cost function given the locations of all nodes at \( P^0_0 \), and vise versa. These two results are the foundation of our iterative motion algorithm.

In the \( k^{th} \) iteration of our motion algorithm, all nodes update their dominant subregions \( W^k_i \) first. They then compute the mass \( M^k_i \) and centroids \( C^k_i \) of the subregions and send them to the sink node. The sink node moves to a new location computed by (13) and broadcasts it to all mobile nodes. In the end, all mobile nodes move to new locations computed by (12).

Remark 6: When both the partition and locations are said to be optimal, it means the cost function reaches a local minimum. The global optimality cannot be guaranteed under our motion control algorithm because of the nonlinearity of the cost function. There may exist multiple local minima. Which local minimum the cost function converging to depends on the initial conditions. How does the cost function reach the global minimum is still an open problem, see [2], [3].

Algorithm 1 The discrete time distributed iterative motion control algorithm for optimal mobile nodes placement serving information dissemination

Require: Iteration \( k \), mobile node \( i \), node location \( p^k_{i-1} \), \( i \in \{0, 1, \cdots, n\} \)
1: All nodes update power cells \( W^k_i \)
2: All nodes compute \( M^k_i \) and \( C^k_i \)
3: All mobile nodes send \( M^k_i \) and \( C^k_i \) to the sink node
4: The sink node moves to \( p^k_0 \) computed by (13)
5: The sink node broadcasts \( p^k_0 \) to all mobile nodes
6: All mobile nodes move to \( p^k_0 \) computed by equation (12)
7: \( k \) = \( k + 1 \)

Theorem 1: If the motion of all nodes obeys Algorithm 1, the cost function \( J_{dis}(P, W) \) decreases monotonically along iterations and eventually converges to a local minimum.

The proof is similar as in [1].

V. SIMULATIONS

In this section, we provide a numerical simulation for seven mobile nodes and one sink node to achieve an optimal service configuration in a unit square area \( \Omega = [0, 1] \times [0, 1] \). All nodes are randomly placed in the area. We assume that other than aggregating and distributing information, the sink node provides service to its dominant subregion just as other mobile nodes do. We run 50 iterations to show that under our distributed motion control algorithm, all nodes eventually reach an optimal service configuration that minimizes the information dissemination cost function \( J_{dis}(P, W) \). We also show that the information dissemination cost function \( J_{dis}(P, W) \) decreases monotonically along iterations.

The information generating density function used in the simulation is a sum of a Gaussian distribution and a positive constant \( \phi(x, y) = 0.1 + 2e^{-0.75(4x-3)^2-0.75(4y-3)^2} \). The constant \( \alpha \) that weighs the cost of information flow between mobile nodes and the sink node is 0.25. Since the algorithm is implemented in discrete time, we set the maximal step of location change for any node in an iteration to 0.01, which is 1/100 the length of the side of \( \Omega \).

An advantage of our algorithm is that the locational optimization does not explicitly depend on the information dissemination function \( s(q, q) \) of any point in the area. It only explicitly depends on the information receiving density which can be easily obtained from statistical results. Based on the above reason, we simulate our control algorithm in such a way that we just let the information receiving function be \( r(x, y) = 0.514 \frac{1}{0.379} \left(3e^{-3(4x-2)^2-3(4y-1)^2} + 0.5e^{-4(4x-1)^2-4(4y-3)^2}\right) \) without knowing \( s(q, q) \) over the area. We need the constant \( 0.514 \frac{1}{0.379} \) to make sure that both integrals of \( \phi(q) \) and \( r(q) \) over \( \Omega \) are equal.

Fig. 2 shows the initial service configuration. The dark area is the place where the sum of information generating and receiving density is high. Fig. 3 shows the final service configuration when the cost function reaches a local minimum. The blue lines in the figure show how the nodes are driven
VI. Conclusion

In this paper, we study the problem of optimal sensor location of information dissemination. We formally model the cost of information dissemination. By interpreting coverage cost as the cost of information aggregation from all points in the area to their assigned mobile node, the coverage with information aggregation problem in [1] can be considered a special case of information dissemination where the sink node is the final destination of all information generated in the area. By defining the information dissemination function, we build a link between the information generating and receiving density function. Furthermore, we formalize the optimal sensor location problem by minimizing the information dissemination cost with respect to both the partition of the area and locations of all nodes. We derive the optimality conditions for the partition of the space and location of mobile nodes. We solve the optimal location problem by applying a discrete time distributed iterative motion control algorithm. Also, we prove that the cost function decreases monotonically along iterations and converges to a local minimum under our motion control algorithm.

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